Impact of Information Asymmetry and Limited Production Capacity on Business Interruption Insurance

Yuan-Mao Kao
Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, United States.
yuan.mao.kao@duke.edu

N. Bora Keskin
Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, United States.
bora.keskin@duke.edu

Kevin Shang
Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, United States.
khshang@duke.edu

We consider a firm that faces disruption risks and can purchase business interruption (BI) insurance from an insurer to guard against the risks. The firm makes demand forecasts, and can put a recovery effort if a disruption occurs; both are unobservable to the insurer. Accordingly, the insurer offers BI insurance to the firm while facing adverse selection and moral hazard. We first find that because of limited production capacity, the firm with a lower demand forecast benefits more from BI insurance than that with a higher demand forecast, and has incentive to pretend to have the higher demand forecast to obtain more profit. We then characterize the optimal insurance contracts to deal with the information asymmetry, and show how the firm’s operational characteristics affect the optimal contracts. We also analyze the cases where the insurer ignores the information asymmetry in designing insurance contracts, and characterize the impact of such ignorance on the firm and the insurer. While the insurer always incurs a profit loss due to ignorance, the insured firm does not necessarily benefit from the insurer’s ignorance.

Key words: Business Interruption Insurance; Moral Hazard; Adverse Selection; Disruption Management

History: May 23, 2018

1. Introduction

Business interruption (BI) insurance is a key financial mitigation strategy in disruption management. When a company faces a disruption, its operations would be suspended, resulting in income loss for the company. In such cases, BI insurance helps companies offset their income losses due to disruptions. For example, Ericsson suffered a substantial loss due to a disruption at its chip supplier in 2000, and subsequently received approximately $200 million dollars from insurance companies as compensation (Norrman and Jansson 2004). Furthermore, BI insurance supports companies in the face of disruptions due to natural disasters, such as hurricanes and fires (see, e.g., McLeod 2006, King 2013). Because BI insurance provides a significant financial support to the insured, many companies are inclined to purchase BI insurance to guard against disruption risks. According
to a survey conducted by Dixon and Stern (2004) for retail businesses in Lower Manhattan, more than half of the businesses own BI insurance. In another survey, conducted in Florida with 252 business owners and managers, almost half of these respondents plan to purchase or have already purchased BI insurance as a proactive action to mitigate income loss from hurricanes (Howe 2011). As indicated by these studies, BI insurance is attractive to companies for mitigating disruption risks in practice.

Despite the interest in BI insurance, the design and implementation of BI insurance involves many challenges due to information asymmetry. First, after a disruption, a company can make an effort to resume business operations; such effort is unobservable to insurers. Purchasing BI insurance could alter a company’s effort on recovery, which in turn affects the compensation for the disruption loss. This effect is known as moral hazard (Hölmstrom 1979). Adachi et al. (2016) provide evidence of moral hazard in the implementation of BI insurance in the 2011 Thailand floods. After the floods, insured companies put less effort on recovery (e.g., reducing the number of workers), resulting in lower production levels and longer suspension times, relative to uninsured companies. Second, a company might have private information (e.g., demand forecasts) that the insurer might not be able to observe. Without complete information on the demand, the insurer may underestimate or overestimate the company’s loss due to the disruption. This may result in overcompensation, which negatively impacts the insurer’s profit. This is known as adverse selection (Mussa and Rosen 1978). Meuwissen et al. (2003) report that BI insurance is common in the agriculture industry, especially in animal husbandry, where information asymmetry naturally arises. For example, farmers have information related to risks of livestock epidemics and mortality rates of livestock, while insurers may not have such information.

Based on these studies, the information asymmetry, either in the form of moral hazard or adverse selection, has an impact on BI insurance. Companies may purchase BI insurance to mitigate disruption risks, and they also make an effort to recover their operations after disruptions; those decisions depend on companies’ operational and informational characteristics. Some operational characteristics, such as capacity, price, and cost, may be observable to insurers when offering BI insurance, but the informational characteristics (e.g., demand forecasts) and companies’ recovery efforts could be kept unobservable to insurers, resulting in information asymmetry. To investigate such impact on both insurers and companies (the insurers’ potential customers), we focus on the following questions: how do a company’s operational characteristics affect information asymmetry on BI insurance? Given the operational considerations, how should an insurer design its BI insurance contracts under information asymmetry? If the insurer ignores information asymmetry when
designing insurance contracts, how does this ignorance affect the insurer’s profit and companies’ interest in BI insurance?

To answer the above questions, we consider an insurer offering BI insurance to a firm over a selling season. The firm faces demand uncertainty but has private information on the magnitude of demand forecasts (i.e., high or low demand). In addition, the firm faces disruption risks during the selling season. To mitigate the disruption risks, the firm can exercise a financial mitigation strategy by purchasing BI insurance in the beginning of the selling season. In particular, the firm pays a premium to the insurer and receives a compensation of a percentage of the revenue loss due to the disruption. The firm can also exercise an operational mitigation strategy by making a recovery effort to reduce its loss from the disruption. The firm maximizes its expected profit via its insurance purchasing and operational effort decisions. The insurer observes neither the firm’s demand forecasts nor the firm’s effort after a disruption. Accordingly, the insurer designs optimal contracts for the firm under adverse selection (due to the unobservable demand forecasts) and moral hazard (due to the unobservable recovery effort) to maximize its profit (see Section 3 for details on our formulation).

We make three contributions to the research on BI insurance under information asymmetry. Our first contribution is concerned with the BI insurance design in the presence of capacity constraints. At first glance, one might intuitively think that a company with a higher demand forecast benefits more from BI insurance and would be willing to pay more for BI insurance, because with a high forecast, this company is supposed to be compensated more for its income loss if a disruption occurs. However, surprisingly, our analysis indicates that a company can benefit more from BI insurance when its demand forecast is lower. This is because insurance reimbursement compensates the lost sales due to the disruption and excludes the lost sales due to the capacity limit. Thus, a firm with a low demand forecast is less likely to reach the capacity limit, meaning that, if a disruption occurs, the lost sales would be mainly due to the disruption. Consequently, the insurance compensation is more valuable to the low-demand firm. Propositions 3 and 4 present the relevant statements for this property: given an insurance contract, the firm with a lower demand forecast benefits more from greater BI insurance coverage and has higher additional profit by owning BI insurance, relative to having a higher demand forecast. This result also implies that if an insurer offers a menu of contracts to a potential insurance customer based on different demand-forecast scenarios, then the customer observing a lower demand forecast may have incentive to pretend to have a higher demand forecast to obtain higher profit from BI insurance.
Second, we show how the design of BI insurance depends on the potential customer’s operational and informational characteristics. For example, regarding operational characteristics, if a firm has more capacity, the lost sales is more likely due to the disruption. Thus, the BI insurance is more valuable to the firm and the insurer can charge more by increasing the insurance coverage and premium to earn more profit. Regarding informational characteristics, if the insurer expects that the potential customer is more likely to have a higher demand forecast, the insurer would increase the premium of the contract designed for the case of a higher demand forecast, but decrease the premium of the contract designed for the case of a lower demand forecast. It is because, in this situation, the insurer tends to earn more profit in the case where the company has a higher demand forecast, but has to forgo some profit if the company has a lower demand forecast; otherwise, this company would pretend to have a higher demand forecast when purchasing the insurance. Propositions 9 to 13 in Section 6.1 provide a detailed analysis of those effects.

Third, we analyze the cases where the insurer ignores the information asymmetry when designing insurance contracts for a customer. In any form of ignorance (e.g., ignoring adverse selection, moral hazard, or both), the insurer would lose the customer with a positive probability due to mispricing the contracts (see Proposition 14), resulting in an expected profit loss (see Proposition 15). Whether the impact of ignoring adverse selection is greater than that of ignoring moral hazard depends on the customer’s operational and informational characteristics. Interestingly, the cost of ignoring both adverse selection and moral hazard might not be the largest. It is because the insurance contracts under any type of ignorance are all mispriced, and the contracts designed when ignoring both effects might end up targeting the customer’s characteristics more closely, relative to ignoring only adverse selection or moral hazard. While a company with a lower demand forecast benefits from information asymmetry, but whether this company can further benefit from the aforementioned ignorance depends on its operational as well as informational characteristics (see Section 7.2 for a detailed numerical investigation).

The remainder of this paper is organized as follows. Section 2 reviews the related literature, and Section 3 describes the model. We analyze the firm’s optimal effort on recovery and its expected profit with and without BI insurance in Section 4, and investigate the insurer’s problem and characterize its optimal contracts in Section 5. In Section 6, we present our analytical insights on the optimal contracts and the impact of ignorance. Section 7 provides the numerical results that illustrate our insights. Lastly, Section 8 includes a summary of the paper.
2. Literature Review

Our study is related to the literature on: (i) business interruption insurance (especially in the context of operations management), (ii) disruption management, and (iii) information asymmetry.

With regard to (i), there is a multitude of law and insurance studies that investigate the practice of BI insurance (see, e.g., Borghesi 1992, Brennan and Conway 2002, Dempsey and Epstein 2002, Lentz and Reeves 2017). These studies primarily focus on the implementation of BI insurance and the processing of claims due to disruptions. By contrast, the operations management literature on BI insurance is relatively limited. Dong and Tomlin (2012) is the first study that jointly considers BI insurance and operational decisions to mitigate disruption risks. They find that BI insurance and operational mitigation are not always substitutes although they both offset the loss from disruptions. Dong et al. (2015) extend the above study and consider the similar framework in a two-stage production chain. Moreover, they consider preparedness actions to lower disruption risks beforehand. Serpa and Krishnan (2016) also study the interaction between preparedness efforts and business insurance in a supply chain with a supplier and an operator. In their study, purchasing business insurance would reduce the proactive efforts on lowering disruption risks. On top of its financial support for the insured companies, business insurance strategically prevents free riding in the preparedness efforts between the supplier and the operator. Zhen et al. (2016) study joint strategies of BI insurance, effort on recovery, and backup supply for a distribution center. Under disruption, the BI insurance compensates the income loss for the distribution center, the backup supply supports the operations, and the effort on recovery reduces recovery time. Unlike the above studies, we take insurer’s product design decisions into account, and investigate how these decisions interact with company’s decisions on mitigating disruption risks. Moreover, our work studies the impact of the information asymmetry on both the insurer and its potential customer.

With respect to (ii), there is a rich literature on supply chain disruption management. We refer readers to the survey by Snyder et al. (2016) for a comprehensive review of that literature. As indicated in this survey, many studies focus on proactive strategies (e.g., sourcing mitigation) to mitigate disruption risks (see, e.g., Anupindi and Akella 1993, Berger et al. 2004, Tomlin 2006). On the other hand, operational contingency (e.g., backup supply) is a common reactive strategy to reduce income loss after disruptions (see Tomlin 2006). There are also some studies that consider revising production schedules after disruptions to satisfy the demand (see Hishamuddin et al. 2012). However, in those studies, recovery time is exogenous and independent of the firm’s actions. In this context, Zhen et al. (2016) consider a recovery decision that can shorten the recovery time after
a disruption. In our work, we study how a company’s recovery effort after a disruption interacts with its insurance purchasing decision and the insurer’s contract design decision.

With regard to (iii), the seminal work by Mussa and Rosen (1978) is a classic reference for adverse selection. In this paper, the authors study how a monopoly designs a price-quality menu for its products to partially discriminate the consumers under incomplete information, showing that the consumer with lower preference for product quality would be offered the product with lower quality under the optimal price-quality menu. The work by Hölmstrom (1979) is a well-known study in modeling moral hazard in a principal-agent relationship. If a principal cannot observe an agent’s actions but observe the agent’s payoff, the principal can design a contract that optimally allocates the payoff to the agent and the principal. The principal’s profit under this contract, however, is still worse than that under the contract when the agent’s actions are observable; this is due to moral hazard. Based on these foundational studies, there are theoretical papers that study adverse selection and moral hazard in insurance (see, e.g., Rothschild and Stiglitz 1976, Shavell 1979, Chade and Schlee 2012). Our analysis indicates that operational considerations can significantly influence the design of BI insurance, separating our work from the above studies: due to the capacity constraints, a company with a lower demand forecast benefits more from BI insurance, and the insurer would offer this company a contract with higher insurance coverage and premium, relative to the contract designed for the case of a high demand forecast.

3. Model
We consider an insurer offering BI insurance to a firm in advance of a selling season. The firm sells a product based on its capacity, and the insurer designs a menu of insurance contracts for the firm. In the following subsections, we introduce the model in terms of the firm and the insurer.

3.1. The Firm’s Problem
The demand for the firm’s product over the selling season is uncertain and given by

\[ D = \mu + \delta + \epsilon, \]  

(3.1)

where \( \mu \) is the firm’s early demand forecast, \( \delta \) is the firm’s forecast update, and \( \epsilon \) is a random demand shock. The early demand forecast \( \mu \) is observable to the firm in the beginning of the selling season (but not observable to the insurer). This can be interpreted as the firm’s initial information on the market conditions in this selling season. During the selling season, the firm obtains further information on the market conditions, updating its initial forecast by \( \delta \), while the random demand
shock $\epsilon$ remains unobservable to the firm. Our modeling approach is to imitate the practice that a firm can obtain a more accurate demand information as it moves onto the selling season.

We assume that there are two possible market conditions for the early demand forecast. The market condition is “high” if $\mu = \mu_H$, and “low” if $\mu = \mu_L$, where $\mu_H > \mu_L > 0$. The forecast update is also assumed to have two possible outcomes. The demand is predicted to increase if $\delta = \delta_H$ and decrease if $\delta = \delta_L$, where $\delta_H > 0 > \delta_L$. Conditional on the early demand forecast $\mu_i$ and the forecast update $\delta_j$, where $i, j \in \{H, L\}$, the firm’s demand variable would be

$$D_{ij} = \mu_i + \delta_j + \epsilon,$$

where $i, j \in \{H, L\}$. (3.2)

In (3.2), $\mu_i + \delta_j$ represents the “updated demand forecast” after observing $\mu_i$ and $\delta_j$. Without loss of generality, we assume the expected value of the forecast update $\delta$ is 0. This is because any non-zero expected value of the forecast update can be subsumed into the early demand forecast, keeping the same distribution of the updated demand forecast. We denote the probability of $\mu = \mu_H$ by $\alpha$ (and hence the probability of $\mu = \mu_L$ is $1 - \alpha$); the probability of $\delta = \delta_H$ is $\beta$ and that of $\delta = \delta_L$ is $1 - \beta$, where $E[\delta] = \beta \delta_H + (1 - \beta) \delta_L = 0$. We assume that $\mu$ and $\delta$ are independent random variables since the forecast update $\delta$ is based on entirely new demand information (relative to the early demand forecast $\mu$). We also assume that $\epsilon$ follows a logistic distribution with parameters $m$ and $\gamma$, where the mean $m = 0$ and the variance is $\frac{1}{3} \pi^2 \gamma^2$. We let $f_{ij}(\cdot)$ and $F_{ij}(\cdot)$ denote the probability density function and the cumulative distribution function of the demand variable $D_{ij}$, respectively. These are given by

$$f_{ij}(x) = \frac{\exp\left(-\frac{x-(\mu_i+\delta_j)}{\gamma}\right)}{\gamma \left[1 + \exp\left(-\frac{x-(\mu_i+\delta_j)}{\gamma}\right)\right]^2},$$

(3.3)

$$F_{ij}(x) = \frac{1}{1 + \exp\left(-\frac{x-(\mu_i+\delta_j)}{\gamma}\right)}.$$

(3.4)

The logistic distribution has been utilized in earlier operations management studies because it can closely approximate the normal distribution and has closed form expressions on its (inverse) cumulative distribution function (Figure 1 shows the comparison between these two distributions). For example, Shang and Song (2006) adopt a logistic distribution to approximate a demand distribution in a serial inventory system. Egri and Vánca (2012) also utilize this distribution as a demand distribution in a newsvendor problem for channel coordination.
Figure 1  Comparison of Logistic and Normal Distribution. The parameters of the normal distribution are $(\mu, \sigma) = (500, 150)$, and those of the approximating logistic distribution are $(m, \gamma) = (\mu, 0.617\sigma) = (500, 92.55)$.

Under normal circumstances, the firm satisfies the demand based on its initial capacity $K$, which is known to the insurer. However, the firm is at risk for disruptions. Once a disruption occurs, all of the firm’s initial capacity becomes unavailable, and thus, the firm would be unable to fulfill any demand, resulting in revenue loss. In addition, the firm incurs a financial cost caused by this unexpected disruption. For example, the firm might have to pay additional cost for external financing or lose the opportunity for investment due to insufficient working capital under disruption. Due to this indirect impact, the firm should take such disruption penalty cost into account (see (4.4) and (4.10) for our definition of the disruption penalty cost, and Dong and Tomlin 2012, Dong et al. 2015 for similar formulations). To mitigate the loss due to the disruption, the firm can take the following operational and financial actions.

With regard to the operational actions, the firm can put an effort into capacity recovery, such as increasing the labor hours in recovery. We assume that the capacity can be resumed immediately with the effort, and the firm can then satisfy the demand by the resumed capacity. However, such effort is costly; for example, the firm may have to pay overtime wages. Thus, the firm has to decide how much effort to put on the recovery, which is a disruption management problem (see Sections 4.1 and 4.2 for details).

With regard to the financial actions, the firm can purchase business interruption (BI) insurance in the beginning of the selling season. With BI insurance, the firm can receive the reimbursement for the revenue loss from disruption. In our model, we consider coinsurance-based BI insurance with a premium $y$ and coverage percentage $s$. That is, if the firm purchases BI insurance, it has to pay $y$ initially but receives $s$ (percentage) times its revenue loss as a compensation under disruption. Given the BI insurance, the firm can assess the value of the insurance and its recovery effort under disruption (see Section 4.2 for details). In the next subsection, we describe the sequence of the events and the firm’s decisions in detail.
3.2. Sequence of Events

In the beginning of the selling season, the insurer offers BI insurance to the firm. Given the insurance, the events and the firm’s decisions are described in the following and illustrated in Figure 2.

**Figure 2**  The Firm’s Decision Process.

1. The firm observes the early demand forecast \( \mu_i \), where \( i \in \{H, L\} \).
2. The firm decides whether to purchase the insurance or not. We let \( l \) denote the purchasing decision, where \( l = I \) represents purchasing and \( l = N \) represents not purchasing.
3. The firm observes the forecast update \( \delta_j \), where \( j \in \{H, L\} \).
4. With probability \( \rho \), a disruption occurs before the firm’s demand is realized. We let \( Z \) denote a binary indicator variable representing the disruption event, where \( Z = 1 \) if a disruption occurs, and \( Z = 0 \) otherwise.
5. If a disruption occurs, the firm chooses its recovery effort level \( e \in [0, 1] \), where the marginal cost of effort per unit capacity is \( c \).
6. The demand \( D \) is realized. In the absence of disruption, the firm satisfies the demand based on its capacity \( K \). Under disruption, the firm fulfills the demand by its resumed capacity \( eK \). In either case, the firm receives a price \( p \) for each unit sold, where \( p > c \).
7. In the end, if a disruption occurs and the firm has insurance, the insurer will compensate the firm for its revenue loss from the disruption in coverage percentage \( s \).

In the disruption management, the effort decision \( e \) represents the percentage of the capacity that the firm decides to recover. Given the effort \( e \), the recovered capacity would be \( eK \), and the firm has to pay the cost of effort \( ceK \). The firm needs to decide the optimal effort under disruption with and without the insurance, and evaluate the expected profits in these two cases. We provide a detailed analysis of the firm’s decisions and profit in Section 4.
3.3. The Insurer’s Problem

The insurer offers the BI insurance with premium $y$ and the coverage percentage $s$ to the firm. We assume that $s \in [\bar{s}, \bar{s}]$, where $0 \leq \bar{s} < \bar{s} \leq 1$. As alluded to earlier, if the firm purchases the insurance, the insurer receives the premium but has to compensate for the firm’s revenue loss under disruption.

The information that the insurer possesses is as follows. The insurer knows that the firm is either a “high-demand” firm (i.e., $\mu = \mu_H$) or a “low-demand” firm (i.e., $\mu = \mu_L$) and the corresponding probability (i.e., $\alpha$), but cannot identify the firm’s type. In accordance with this, we hereafter use high-demand firm and low-demand firm to represent the cases in which the firm has a high and low early demand forecast, respectively. In addition, the insurer knows that the firm will conduct a demand forecast update $\delta$ during the selling season and the corresponding probability (i.e., $\beta$), but does not know exactly the update value $\delta$.

Because there are two possible early demand forecasts of this firm, the insurer offers a menu of the insurance contracts, $(y_H, s_H)$ and $(y_L, s_L)$, tailored to the firm’s forecasts. Given that the insurer only knows the probability of each forecast but cannot observe the exact value of the firm’s forecast, it is unable to offer a “full-information” insurance contract. Consequently, this is an adverse selection problem due to the firm’s private information of $\mu$.

The compensation for the firm’s revenue loss depends on the firm’s recovery effort, which depends on the updated demand forecast $\mu + \delta$. Although the insurer may identify the firm’s early demand forecast after the firm’s purchasing decision, the insurer does not know the firm’s forecast update $\delta$. As a result, the insurer is unable to predict the firm’s exact recovery effort. This results in a moral hazard problem caused by the firm’s hidden action (namely, the recovery effort). Accordingly, the insurer designs the contracts $(y_H, s_H)$ and $(y_L, s_L)$ to maximize its profit and mitigate the impact of both moral hazard and adverse selection. The analysis of the insurer’s decisions and profit is provided in Section 5.

4. Analysis of the Firm’s Problem

According to the decision process in Figure 2, the firm can optimize its decisions via a backward solution. In this backward solution, we first solve for the optimal effort under disruption in each scenario, and then evaluate the firm’s expected profit with and without BI insurance.

In this section, we first analyze the firm’s expected profit without BI insurance (with purchasing decision $l = N$), and then conduct a similar analysis for the case with BI insurance ($l = I$). Finally, we provide some properties of the firm’s optimal effort and the firm’s expected profit with respect to the insurance contract $(y, s)$.
4.1. The Firm’s Profit without BI Insurance

Given the updated demand forecast $\mu_i + \delta_j$, where $i, j \in \{H, L\}$, there are two possible outcomes of the firm’s expected profit. If no disruption occurs ($Z = 0$) during the selling season (i.e., under normal operations), the firm’s expected profit without BI insurance is:

$$\pi_{ij0}(K) = pE[\min(D_{ij}, K)] = p\{K + \mu_i + \delta_j - \gamma w(\mu_i, \delta_j)\}, \quad (4.1)$$

where

$$w(\mu_i, \delta_j) = \ln \left[\exp \left(\frac{K}{\gamma}\right) + \exp \left(\frac{\mu_i + \delta_j}{\gamma}\right)\right] \quad (4.2)$$

and $E[\cdot]$ denotes the expectation with respect to the random demand shock $\epsilon$. Because no recovery decision needs to be made in this case, the firm’s expected profit depends on its capacity $K$.

If a disruption happens ($Z = 1$), the firm faces a disruption management problem and has to decide on the recovery effort. We let $\pi_{ij1}(K, e)$ denote the expected profit function in this case, which is given by

$$\pi_{ij1}(K, e) = -ceK + pE[\min(D_{ij}, eK)] - C_{ij}^N(K, e), \quad (4.3)$$

where

$$C_{ij}^N(K, e) = \lambda \left\{ pE[\min(D_{ij}, K)] + ceK - pE[\min(D_{ij}, eK)] \right\}. \quad (4.4)$$

The expected profit in (4.3) consists of three terms. The first and second terms correspond to the firm’s cost of the effort and expected revenue based on its resumed capacity $eK$, respectively. The third term, $C_{ij}^N(K, e)$, represents the disruption penalty cost, given in (4.4). In that equation, $\lambda$ is the unit penalty cost (where $\lambda > 0$), and the term in the curly brackets is the profit loss under disruption relative to the profit under normal operations. This problem can be considered as a variant of the capacitated newsvendor problem, and the optimal effort decision $\hat{e}_{ij}$ without insurance is

$$\hat{e}_{ij} = \begin{cases} \frac{1}{K}F_{ij}^{-1}\left(\frac{p-c}{p}\right) = \frac{1}{K} \left[\mu_i + \delta_j + \gamma \ln \left(\frac{p-c}{e}\right)\right] & \text{if } F_{ij}^{-1}\left(\frac{p-c}{p}\right) < K, \\ 1 & \text{otherwise}. \end{cases} \quad (4.5)$$

In (4.5), once the effort level satisfying the unconstrained first-order condition exceeds 1, the optimal effort becomes 1 due to the constraint $0 \leq e \leq 1$.

Based on the optimal effort, the firm can evaluate the optimal expected profit under disruption. With slight abuse of notation, we let $\pi_{ij1}^N(K)$ be the optimal expected profit in this case, where

$$\pi_{ij1}^N(K) = \pi_{ij1}^N(K, \hat{e}_{ij}). \quad (4.6)$$
Given the expected profits in (4.1) and (4.6), we let $E_{\delta,Z}[\pi_i^N(K)]$ denote the firm’s expected profit without BI insurance when $\mu = \mu_i$; that is,

$$E_{\delta,Z}[\pi_i^N(K)] = \beta \left\{ \rho \pi_{iH1}^N(K) + (1 - \rho) \pi_{iH0}^N(K) \right\} + (1 - \beta) \left\{ \rho \pi_{iL1}^N(K) + (1 - \rho) \pi_{iL0}^N(K) \right\},$$

where $E_{\delta,Z}[\cdot]$ denotes the expectation with respect to the forecast update $\delta$ and the disruption indicator $Z$.

### 4.2. The Firm’s Profit with BI Insurance

Given the updated demand forecast $\mu_i + \delta_j$, where $i, j \in \{H, L\}$, if no disruption occurs, the firm’s expected profit with BI insurance is

$$\pi_{ij0}(K, y, s) = -y + pE_e[\min(D_{ij}, K)] = -y + p \{K + \mu_i + \delta_j - \gamma w(\mu_i, \delta_j)\}.$$  (4.8)

Comparing this to the expected profit without insurance in the absence of a disruption in (4.1), we note that both include the same expected sales revenue. But in (4.8), there is an additional purchasing cost $y$ that the firm pays for the BI insurance in this case.

If a disruption occurs, similarly the firm has to decide the recovery effort. We let $\pi_{ij1}(K, e, y, s)$ denote the expected profit function in this case, which is

$$\pi_{ij1}(K, e, y, s) = -y - ceK + pE_e[\min(D_{ij}, eK)] + spE_e[\min(D_{ij}, K) - \min(D_{ij}, eK)] - C_{ij}(K, e, y, s),$$

where

$$C_{ij}(K, e, y, s) = \lambda \left\{ pE_e[\min(D_{ij}, K)] + ceK - pE_e[\min(D_{ij}, eK)] \right\}.$$  (4.10)

In (4.9), in addition to the revenue the firm earns based on the resumed capacity, it also receives the reimbursement for its revenue loss caused by the disruption. The loss is defined to be the difference between the revenue under the normal operations and that under disruption. Given the coverage percentage $s$, the reimbursement amount is $spE_e[\min(D_{ij}, K) - \min(D_{ij}, eK)]$ in (4.9). In addition, the disruption penalty in this case is expressed in (4.10).

With coverage percentage $s$, the optimal recovery effort $e_{ij}^*(s)$ that maximizes (4.9) is

$$e_{ij}^*(s) = \begin{cases} \frac{1}{K} F_{ij}^{-1} \left( \frac{p(1-s)-c}{p(1-s)} \right) = \frac{1}{K} \left[ \mu_i + \delta_j + \gamma \ln \left( \frac{p(1-s)-c}{c} \frac{p(1-s)}{p(1-s)} \right) \right] & \text{if } F_{ij}^{-1} \left( \frac{p(1-s)-c}{p(1-s)} \right) < K, \\ 1 & \text{otherwise.} \end{cases}$$

(4.11)

The derivation of (4.11) is in Appendix A. As argued earlier for $\hat{e}_{ij}$, the optimal effort $e_{ij}^*(s)$ cannot exceed 1, and we are interested in the case of a positive recovery effort. To rule out cases in which
the firm puts no effort into recovery, one needs to have $s < 1 - \frac{\delta}{\mu + \delta} \left[ 1 + \exp \left( -\frac{\mu + \delta}{\gamma} \right) \right]$ for all $i$ and $j$. Thus, we further assume that the upper bound on $s$, namely $\bar{s}$, is less than $1 - \frac{\delta}{\mu + \delta} \left[ 1 + \exp \left( -\frac{\mu + \delta}{\gamma} \right) \right]$.

Letting $\pi_{ij1}(K, y, s)$ denote the optimal expected profit in this case, we have

$$\pi_{ij1}(K, y, s) = \pi_{ij1}(K, e^*_ij(s), y, s).$$

(4.12)

Given the expected profits in (4.8) and (4.12), we let $E_{\delta,Z}[\pi_i^I(K, y, s)]$ denote the firm’s expected profit with BI insurance when $\mu = \mu_i$:

$$E_{\delta,Z}[\pi_i^I(K, y, s)] = \beta \{ \rho \pi_{ih1}^I(K, y, s) + (1 - \rho)\pi_{ih0}^I(K, y, s) \} + (1 - \beta) \{ \rho \pi_{il1}^I(K, y, s) + (1 - \rho)\pi_{il0}^I(K, y, s) \}.$$  \hspace{1cm} (4.13)

Comparing the expected profit without and with BI insurance in (4.7) and (4.13), respectively, the firm makes its BI insurance purchasing decision: it would purchase the BI insurance $(y, s)$ if the expected profit with this insurance is greater than or equal to that without insurance.

**4.3. Properties of the Firm’s Profit and Optimal Effort**

In this subsection, we establish the key properties of the firm’s expected profit and optimal effort. In particular, we focus on how the features of the insurance contract $(y, s)$ affect the firm’s expected profit and decisions. The proofs of said properties are provided in Appendix B.

Our first result shows how the optimal effort with BI insurance, namely $e^*_ij(s)$, is affected by the coverage percentage $s$.

**Proposition 1.** Given $\mu_i + \delta_j$, where $i, j \in \{H, L\}$, the firm’s optimal recovery effort with BI insurance, $e^*_ij(s)$, is non-increasing and concave in $s$, and $e^*_ij(s) \leq \hat{e}_ij$ for all $s$.

Proposition 1 states that if the firm purchases the BI insurance, then as $s$ increases, the firm would put less effort on the recovery, and the decay rate of the effort in $s$ is greater when $s$ becomes higher. Moreover, without any coverage ($s = 0$), the firm’s optimal effort $e^*_ij(0)$ is equal to $\hat{e}_ij$ because no compensation is made to help the firm mitigate its disruption loss. This proposition shows that the firm’s insurance strategy and recovery effort are substitutes: the firm reduces its recovery effort with BI insurance. Figure 3 illustrates this property in a numerical example.

In our next result, we analyze how the firm’s expected profit with BI insurance, $E_{\delta,Z}[\pi_i^I(K, y, s)]$, depends on $s$.

**Proposition 2.** Given $\mu_i + \delta_j$, where $i, j \in \{H, L\}$, and $K > 0$, the firm’s expected profit with BI insurance, $E_{\delta,Z}[\pi_i^I(K, y, s)]$, is non-decreasing and convex in $s$. 


Proposition 2 shows that the firm’s marginal benefit from the coverage \( s \) is non-decreasing in \( s \). Because higher \( s \) is more desirable to the firm, this induces the insurer to provide insurance with positive coverage to attract the firm.

Our next goal is to characterize how the firm’s benefit from the coverage percentage \( s \) depends on the early demand forecast. At first glance, one might think that the high-demand firm must benefit more from BI insurance because it has higher expected demand and revenue. That is, the high-demand firm has more to lose than the low-demand firm under disruption, meaning that BI insurance must compensate more for the high-demand firm. However, in the following propositions, we show that the low-demand firm in fact benefits more from BI insurance.

**Proposition 3.** Given \( \mu_i + \delta_j \), where \( i, j \in \{H, L\} \), and \( K > 0 \), the marginal profit of the firm with BI insurance in \( s \), \( \frac{d}{ds}E_{\Delta,Z}[\pi_i^*(K, y, s)] \), is non-increasing in \( \mu_i \).

In the economics literature on adverse selection, the single-crossing property is a key condition commonly used in designing a menu of vertically differentiated products. This property states that the customer’s marginal utility in product quality is non-negative and non-decreasing in its private signal (usually referred to as “type”). In our model, the firm’s type corresponds to the early demand forecast \( \mu \), and the product quality corresponds to the coverage percentage \( s \). According to Propositions 2 and 3, in our setting, the firm’s marginal profit per coverage percentage \( s \) is still non-negative, but it is non-increasing in \( \mu_i \) (rather than being non-decreasing). Based on this opposite direction of monotonicity shown in Proposition 3, we refer to this property as the “reverse” single-crossing property. It is perhaps worth emphasizing that, under this reverse single-crossing property, the indifference curves of the firm with two possible forecasts still cross only once, and the adjective “reverse” points to the result that the low-demand firm has higher marginal profit per \( s \).
Based on the reverse single-crossing property, the low-demand firm benefits more from BI insurance as \(s\) increases, which might appear counter-intuitive. To explain this result, we focus on two key elements: recovery effort and capacity.

First, let us consider a hypothetical scenario in which the firm cannot make a recovery effort and must rely on BI insurance to guard against disruptions. In such a case, \(\min(D_{ij}, 0) = 0\) and the high-demand firm benefits more from BI insurance. However, this hypothetical scenario neglects the firm’s capability to make a recovery effort. In our model where recovery effort is allowed, the firm can use its recovery effort as a substitute for insurance coverage, and the firm’s capacity \(K\) moderates the substitution between these two levers. Note that the insurance reimbursement excludes the sales revenue lost due to the original capacity limit \(K\), simply because these sales would have been lost regardless of disruption. This means that the firm’s capacity constraint truncates the insurance reimbursement. Said truncation is more pronounced for the high-demand firm, since demand is more likely to exceed the capacity in the high-demand scenario. Based on this, the high-demand firm would expect a smaller revenue loss due to disruption. As a result, the high-demand firm would value insurance coverage less than the low-demand firm would, and accordingly, the high-demand firm’s preference for recovery effort (over insurance coverage) would be stronger.

Roughly speaking, the high-demand firm has a stronger preference for “insuring itself” via its own recovery efforts, knowing that it will make a substantial recovery effort in case of a disruption. In contrast, the low-demand firm has a stronger preference to purchase insurance from the insurer because its recovery effort would be more limited. Figure 4 illustrates the properties presented in Propositions 2 and 3.

![Figure 4](image_url)

**Figure 4**  **The Firm’s Expected Profit.** The problem parameters are \(y = 0\), \(K = 2000\), \(\mu_H = 1800\), \(\mu_L = 1600\), \(\beta = 0.4\), \(\delta_H = 300\), \(\delta_L = -200\), \(\rho = 0.2\) \(\gamma = 150\), \(p = 10\), \(c = 4\), and \(\lambda = 4\).

Based on Proposition 3, the following result shows that given the same insurance contract, the low-demand firm has higher net profit from the BI insurance (the firm’s expected profit with BI insurance minus that without BI insurance).
Proposition 4. Given $\mu_i + \delta_j$, where $i, j \in \{H, L\}$, $K > 0$ and the insurance contract $(y, s)$, the firm’s net profit from the insurance, $E_{\delta,Z}[^tI_i(K, y, s)] - E_{\delta,Z}[^tN_i(K)]$, is non-increasing with respect to the early demand forecast $\mu_i$.

According to Proposition 4, if the insurer only offers one insurance contract which exploits all the high-demand firm’s net profit from the insurance, this contract is still beneficial to the low-demand firm. In contrast, if the insurer only offers one contract which exploits all the low-demand firm’s net profit from insurance, the high-demand firm would never benefit from that contract. Hence, the low-demand firm has incentive to pretend to have a high early demand forecast to achieve higher profit through the BI insurance offered to the high-demand firm.

5. Analysis of the Insurer’s Problem

In this section, we formulate and simplify the insurance design problem under adverse selection and moral hazard, and then we characterize the optimality of the insurer’s profit based on the insurance design model.

5.1. Problem Simplification

Recalling Section 3.3, we note that the insurer aims to design a menu of insurance contracts $(y_H, s_H)$ and $(y_L, s_L)$ to maximize its profit. Thus, the insurance design problem is as follows.

$$\max_{y_H, s_H, y_L, s_L} \left\{ y_H - \rho \left( \beta s_H p E_s \left[ \min (D_{HH}, K) - \min (D_{HH}, e^*_H(s_H)K) \right] \right) + (1 - \beta) s_H p E_s \left[ \min (D_{HL}, K) - \min (D_{HL}, e^*_H(s_H)K) \right] \right\}$$

$$+ (1 - \alpha) \left\{ y_L - \rho \left( \beta s_L p E_s \left[ \min (D_{LH}, K) - \min (D_{LH}, e^*_L(s_L)K) \right] \right) + (1 - \beta) s_L p E_s \left[ \min (D_{LL}, K) - \min (D_{LL}, e^*_L(s_L)K) \right] \right\}$$

s.t.

$$E_{\delta,Z}[^tI_H(K, y_H, s_H)] \geq E_{\delta,Z}[^tI_L(K, y_L, s_L)] \quad \text{(IC-HL)}$$

$$E_{\delta,Z}[^tI_L(K, y_L, s_L)] \geq E_{\delta,Z}[^tI_H(K, y_H, s_H)] \quad \text{(IC-LH)}$$

$$E_{\delta,Z}[^tI_H(K, y_H, s_H)] \geq E_{\delta,Z}[^tN_H(K)] \quad \text{(IR-H)}$$

$$E_{\delta,Z}[^tI_L(K, y_L, s_L)] \geq E_{\delta,Z}[^tN_L(K)] \quad \text{(IR-L)}$$

We let $R(y_H, s_H, y_L, s_L)$ denote the objective function in (5.1), where the insurer receives the premium (either $y_H$ or $y_L$) from the firm and compensates to the firm if a disruption occurs. There are two kinds of constraints in the above insurance design problem. The individual rationality (IR) constraints ensure that the insurance contracts should be desirable for the firm (as opposed to
operating without insurance). The incentive compatibility (IC) constraints ensure that the firm chooses the contract tailored to its early demand forecast.

In typical adverse selection models, the constraints (IC-LH) and (IR-H) are redundant and the constraints (IC-HL) and (IR-L) are binding under the optimal contracts due to the single-crossing property. Because of the reverse single-crossing property in Propositions 3 and 4, we have the following properties of the constraints under the optimal contracts.

**PROPOSITION 5.** In the optimal solution to the insurer’s problem in (5.1), the constraints (IC-HL), (IR-L) are redundant, and the constraints (IC-LH), (IR-H) are binding.

Based on this proposition, the insurance design problem can be simplified as follows:

\[
\max_{y_H, s_H, y_L, s_L} \alpha \left\{ y_H - \rho \left( \beta s_H p E_r \left[ \min (D_{HH}, K) - \min (D_{HH}, e_{HH}(s_H) K) \right] \right) + (1 - \beta)s_H p E_r \left[ \min (D_{HL}, K) - \min (D_{HL}, e_{HL}(s_H) K) \right] \right\} \\
+(1 - \alpha) \left\{ y_L - \rho \left( \beta s_L p E_r \left[ \min (D_{LL}, K) - \min (D_{LL}, e_{LL}(s_L) K) \right] \right) + (1 - \beta)s_L p E_r \left[ \min (D_{LL}, K) - \min (D_{LL}, e_{LL}(s_L) K) \right] \right\} \tag{5.2}
\]

s.t.

\[
E_{\delta, Z}[\pi_L^I(K, y_L, s_L)] = E_{\delta, Z}[\pi_L^I(K, y_H, s_H)] \tag{IC-LH},
\]

\[
E_{\delta, Z}[\pi_H^I(K, y_H, s_H)] = E_{\delta, Z}[\pi_H^N(K)] \tag{IR-II}.
\]

According to the two binding constraints (IC-LH) and (IR-II), the premiums \(y_H\) and \(y_L\) can be expressed as functions of coverage percentages \(s_H\) and \(s_L\). As a result, the model can be further simplified into an optimization problem without IC and IR constraints. We let \(y_H = g_H(s_H)\) and \(y_L = g_L(s_H, s_L)\), where:

\[
g_H(s_H) = \rho (1 + \lambda) E_\delta \left[ (p(1 - s_H) - c) \left[ \mu_H + \delta + \gamma \ln \left( \frac{p(1 - s_H) - c}{e} \right) \right] - p(1 - s_H) \gamma \ln \left( \frac{p(1 - s_H)}{e} \right) + ps_H \left[ \mu_H + \delta + K - \gamma w(\mu_H, \delta) \right] - \{(p - c) \left[ \mu_H + \delta + \gamma \ln \left( \frac{p(1 - s_H)}{e} \right) \right] - p \gamma \ln \left( \frac{p}{e} \right) \} \right], \tag{5.3}
\]

\[
g_L(s_H, s_L) = g_H(s_H) + \rho (1 + \lambda) E_\delta \left[ (p(1 - s_L) - c) \left[ \mu_L + \delta + \gamma \ln \left( \frac{p(1 - s_L) - c}{e} \right) \right] - p(1 - s_L) \gamma \ln \left( \frac{p(1 - s_L)}{e} \right) + ps_H \left[ \mu_L + \delta + K - \gamma w(\mu_L, \delta) \right] - \{(p - c) \left[ \mu_L + \delta + \gamma \ln \left( \frac{p(1 - s_L)}{e} \right) \right] - p \gamma \ln \left( \frac{p}{e} \right) \} \right], \tag{5.4}
\]

Substituting \(y_H = g_H(s_H)\) and \(y_L = g_L(s_H, s_L)\) into the insurer’s profit function in (5.2), the objective function can be expressed in terms of the coverage percentages \(s_H\) and \(s_L\). Accordingly, we let
\( R(s_H, s_L) \) denote this new objective function. We first find the optimal solution to this problem to characterize the optimal coverage percentages \( s_H^* \) and \( s_L^* \), and then use (5.3)-(5.4) to obtain the optimal premiums \( y_H^* \) and \( y_L^* \).

### 5.2. Properties of the Insurer’s Optimal Contracts

To find the optimal contracts for the insurer, we first optimize the profit function \( R(s_H, s_L) \). By examining the Hessian matrix, we note that \( \frac{\partial^2 R}{\partial s_H \partial s_L} \) is positive. If these inequalities do not hold, the firm may prefer not purchasing insurance and instead recovering all of its capacity in the event of a disruption. Moreover, because the low-demand firm typically has incentive to choose \((y_H, s_H)\), the insurer has to forgo some profit in the contract \((y_L, s_L)\) to prevent such deviation, which reduces the insurer’s profit. Because of this negative impact, the insurer would offer \((y_H, s_H)\) only if the profit from this contract exceeds the profit forgone in \((y_L, s_L)\); inequality (5.5) corresponds to this condition. In summary, these conditions ensure that the insurer can profitably serve the firm.

The next proposition characterizes the optimal coverages \( s_H^* \) and \( s_L^* \).

**PROPOSITION 6.** If \( K > \mu_L + \delta_H + \gamma \ln \left( \frac{p-c}{c} \right) \), then the insurer’s profit \( R(s_H, s_L) \) is quasi-concave in \( s_L \) and the marginal profit at \( s_L = 0 \) is positive. If \( K > \mu_H + \delta_H + \gamma \ln \left( \frac{p-c}{c} \right) \) and

\[
pp \left\{ (1-\alpha)(1+\lambda)\gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)] + \alpha \lambda \left[ K + \gamma \ln \left( \frac{p}{p-c} \right) - \gamma E_\delta [w(\mu_H, \delta)] \right] \right\} \geq 0, \tag{5.5}
\]

then \( R(s_H, s_L) \) is quasi-concave in \( s_H \) and the marginal profit at \( s_H = 0 \) is positive, where \( E_\delta[\cdot] \) denotes the expected value with respect to \( \delta \).

The conditions in this proposition are interpreted as follows. First, the inequalities \( K > \mu_i + \delta_H + \gamma \ln \left( \frac{p-c}{c} \right), \) for \( i \in \{H, L\} \), ensure that the contracts \((y_H, s_H)\) and \((y_L, s_L)\) are attractive to the high-demand and low-demand firms, respectively. If these inequalities do not hold, the firm may prefer not purchasing insurance and instead recovering all of its capacity in the event of a disruption. Moreover, because the low-demand firm typically has incentive to choose \((y_H, s_H)\), the insurer has to forgo some profit in the contract \((y_L, s_L)\) to prevent such deviation, which reduces the insurer’s profit. Because of this negative impact, the insurer would offer \((y_H, s_H)\) only if the profit from this contract exceeds the profit forgone in \((y_L, s_L)\); inequality (5.5) corresponds to this condition. In summary, these conditions ensure that the insurer can profitably serve the firm.

The next proposition characterizes the optimal coverages \( s_H^* \) and \( s_L^* \).

**PROPOSITION 7.** If \( K > \mu_H + \delta_H + \gamma \ln \left( \frac{p-c}{c} \right) \) and (5.5) hold, then \( s_H^* = \min \{s_H^*, \bar{s}\} \) and \( s_L^* = \min \{s_L^*, \bar{s}\} \), where \( s_H^* \) and \( s_L^* \) satisfy the following first-order optimality conditions

\[
(1-\alpha)pp \left\{ (1+\lambda)\gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)] - \gamma E_\delta [w(\mu_H, \delta)] \right\} = 0, \tag{5.6}
\]

\[
(1-\alpha)pp \left\{ \lambda \left[ K + \gamma \ln \left( \frac{p}{p(1-s_H^*\bar{s})-c} \right) - \gamma E_\delta [w(\mu_H, \delta)] \right] - \gamma E_\delta [w(\mu_H, \delta)] \right\} = 0. \tag{5.7}
\]
Note that if the insurer’s profit function satisfies the conditions in Proposition 6, then each of its univariate cross sections is either a quasi-concave function with a unique stationary point or an increasing convex function. Based on this, Proposition 7 states that the optimal insurance coverages are \( s^*_H = \min\{s'_H, \bar{s}\} \) and \( s^*_L = \min\{s'_L, \bar{s}\} \). On the other hand, if said conditions are not satisfied, then zero coverage becomes another candidate for the optimal solution, which implies not offering the contract to the firm (in Appendix B, we provide a few examples for the insurer’s profit function when the aforementioned conditions do not hold). Since the case of positive insurance coverage is the focus of our study, we suppose throughout the sequel that \( K > \mu_H + \delta_H + \gamma \ln \left( \frac{p-c}{c} \right) \) and (5.5) hold, unless otherwise stated.

It is perhaps worth noting that, although Proposition 7 characterizes the optimal insurance coverages in the first-order conditions (5.6) and (5.7), there is no closed-form solution for \( s'_H \) and \( s'_L \). Accordingly, we will employ these first-order conditions to derive analytical insights on the optimal contracts \((y^*_H, s^*_H)\) and \((y^*_L, s^*_L)\). To that end, we first compare the contracts \((y^*_H, s^*_H)\) and \((y^*_L, s^*_L)\) in the next proposition.

**Proposition 8.** Given that the optimal contracts \((y^*_H, s^*_H)\) and \((y^*_L, s^*_L)\) have positive insurance coverages, we have \( s^*_L \geq s^*_H \) and \( y^*_L \geq y^*_H \).

Proposition 8 establishes that, since the low-demand firm benefits more from the insurance coverage (see Propositions 3 and 4), the insurer would provide a contract with higher coverage and higher premium to the low-demand firm, relative to the high-demand firm.

### 6. Analytical Insights

In this section, we first show how the parameters in our model affect the optimal contracts \((y^*_H, s^*_H)\) and \((y^*_L, s^*_L)\). After that, we discuss the cases where the insurer ignores different kinds of information asymmetry, and analyze how such ignorance affects the insurer’s profit and the firm’s decision and benefit from the BI insurance.

#### 6.1. Effects of Problem Parameters on the Optimal Contracts

As argued in Section 5.2, the optimal coverages \( s^*_H \) and \( s^*_L \) may be 0 or \( \bar{s} \) in general. Thus, the optimal coverage offering may be characterized by a corner solution that is insensitive to changes in the problem parameters. To study the optimal contracts that are sensitive to changes in problem parameters, we focus on the case where \( s^*_H = s'_H \) and \( s^*_L = s'_L \) in Proposition 7. The following propositions characterize the effects of different model parameters on the optimal contract features.

**Proposition 9.** As \( \alpha \) (i.e., the probability of \( \mu = \mu_H \)) increases, \( s^*_H \) and \( y^*_H \) increase, \( s^*_L \) remains the same, but \( y^*_L \) decreases.
We note that, as $\alpha$ increases, the impact of adverse selection by the low-demand firm becomes smaller, and the insurer’s profit is more likely to come from the high-demand firm. The insurer would then increase the insurance coverage and premium to better target the high-demand firm. In addition, to prevent the low-demand firm from choosing the high-demand contract, the insurer needs to keep the low-demand contract attractive to the low-demand firm. As indicated in (5.7), the coverage of the low-demand contract does not depend on $\alpha$. Thus, to keep this contract attractive, the insurer would decrease its premium.

**Proposition 10.** As the initial capacity $K$ increases, $s^*_H$, $y^*_H$, $s^*_L$ and $y^*_L$ increase.

When $K$ increases, the reimbursement for the revenue loss becomes less affected by capacity limitations $K$. Accordingly, the insurance becomes more attractive to the firm, and the insurer can increase the insurance coverage and premium in each contract to earn more profit.

**Proposition 11.** As $\mu_H$ increases, $s^*_H$ and $y^*_H$ decrease, $s^*_L$ remains the same, but $y^*_L$ decreases. As $\mu_L$ increases, $s^*_H$ and $y^*_H$ increase, whereas $s^*_L$ and $y^*_L$ decrease.

We note that, when $\mu_H$ increases, the high-demand firm’s benefit from the insurance becomes smaller because said benefit is more likely to be affected by the limited capacity. In this case, the insurer targets the high-demand firm with smaller $s^*_H$ and $y^*_H$. Moreover, increasing $\mu_H$ makes the difference between the two demand forecasts greater, amplifying the impact of adverse selection. The insurer then has to lower the premium $y^*_L$ to prevent the deviation of the low-demand firm.

Conversely, when $\mu_L$ increases, the low-demand firm’s benefit from the insurance becomes less; thus, the insurer would decrease both $s^*_L$ and $y^*_L$. Because the difference between two demand forecasts of the firm becomes smaller in this case, $y^*_H$ and $s^*_H$ in the optimal contract for the high-demand firm both increase due to the reduced impact of adverse selection.

**Proposition 12.** As the disruption probability $\rho$ increases, $s^*_H$ and $s^*_L$ remain the same, but $y^*_H$ and $y^*_L$ increase.

As $\rho$ increases, the value of the insurance for the firm becomes higher because the firm is more likely to face a disruption. Thus, the firm is willing to pay more on the insurance, and the insurer increases the premiums $y^*_H$ and $y^*_L$ to achieve a higher profit. The coverage percentages $s^*_H$ and $s^*_L$ are designed for the compensation after a disruption occurs; hence, they are not affected by the changes in $\rho$.

**Proposition 13.** As the disruption penalty cost $\lambda$ increases, $s^*_H$, $y^*_H$, $s^*_L$ and $y^*_L$ increase.
When the disruption penalty cost $\lambda$ increases, the firm incurs higher cost in the event of a disruption. To mitigate the cost, the firm would rely more on BI insurance, and accordingly, the insurer increases the coverage percentages $s_H^*, s_L^*$ and the premiums $y_H^*, y_L^*$. 

In summary, Proposition 9 to 13 describe how the firm’s operational and informational characteristics affect the optimal contracts. For operational characteristics, we note that the firm relies more on BI insurance when the benefit of BI insurance is less affected by the capacity constraints, or when the financial penalty of profit loss is greater. In those cases, the insurer would offer higher insurance coverage and a higher premium to the firm. Moreover, if the firm is more likely to encounter a disruption, then BI insurance would be more valuable, and the insurer would charge higher premium to the firm. For informational characteristics, note that the impact of adverse selection is less when the difference between high and low demand forecasts is smaller, or the firm is more likely to make a high demand forecast. In such cases, the insurer would increase the insurance coverage and premium to better target the high-demand firm.

The optimal contract terms do not necessarily have monotonic relationships with other parameters. With regard to $\beta$, $\delta_H$, and $\delta_L$ (i.e., the parameters pertaining to forecast updating), we note that their impacts are interdependent since $E[\delta] = \beta\delta_H + (1 - \beta)\delta_L = 0$, and we numerically show that the optimal contracts do not monotonically change in those parameters. Moreover, we also provide numerical examples to show the non-monotonic relationship between the optimal contract terms and the parameters $\gamma$, $p$ and $c$. These numerical illustrations are provided in Appendix C.

6.2. Contracts when Ignoring Information Asymmetry

In this subsection, we investigate the insurance contracts where the insurer ignores information asymmetry. We also provide analytical results about the impact of ignorance on the insurer’s actual profit and the firm’s decisions.

There are two types of the information asymmetry in our problem: adverse selection and moral hazard. Accordingly, there are three types of ignorance: ignoring adverse selection (IA), ignoring moral hazard (IM), and ignoring both (IB). Table 1 shows different types of ignorance and the notation for the corresponding insurance contracts.

<table>
<thead>
<tr>
<th>Adverse Selection</th>
<th>Moral Hazard</th>
<th>Ignore</th>
<th>Not Ignore</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ignore</td>
<td>$(y_{IB}^{IA}, s_{IB}^{IA})$</td>
<td>$(y_{IA}^{IA}, s_{IA}^{IA})$</td>
<td></td>
</tr>
<tr>
<td>Not Ignore</td>
<td>$(y_{IB}^{IA}, s_{IB}^{IA})$, $(y_{L}^{IA}, s_{L}^{IA})$</td>
<td>$(y_{IA}^{IA}, s_{IA}^{IA})$, $(y_{L}^{IA}, s_{L}^{IA})$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 Types of Ignorance and Corresponding Contracts
As indicated in Table 1, if the insurer ignores neither adverse selection nor moral hazard, then the resulting optimal contracts \((y^*_H, s^*_H)\) and \((y^*_L, s^*_L)\) are the ones studied in previous sections. As explained earlier, in our setting, adverse selection stems from the fact that the firm has private demand information, which is unobservable to the insurer. If the insurer ignores adverse selection, it assumes that the firm’s early demand information is the same as the insurer’s. That is, the firm’s early demand forecast is equal to the expected demand, \(\bar{\mu} = \alpha \mu_H + (1 - \alpha) \mu_L\), and we denote the subsequent demand variable in this case by \(D_{MJ} = \bar{\mu} + \delta_j + \epsilon, j \in \{H, L\}\). Because the insurer incorrectly assumes that there is only one early demand forecast of firm, it would offer only one contract \((y^{IA}, s^{IA})\) to the firm. Now, note that the moral hazard in our setting is due to different forecast updates, which subsequently results in different hidden actions regarding the firm’s recovery effort. If the insurer ignores moral hazard, it assumes that there is no forecast update \(\delta\), which would make the firm’s recovery efforts perfectly predictable (without changing the expected demand). In this case, the insurer would offer two contracts \((y^{IM}_H, s^{IM}_H), (y^{IM}_L, s^{IM}_L)\) to the firm only based on the early demand forecast. Lastly, if the insurer ignores both adverse selection and moral hazard, then it assumes that the firm has the early demand forecast \(\bar{\mu}\) and no forecast update. Hence, the demand variable is assumed to be \(D_M = \bar{\mu} + \epsilon\), and the insurer would offer only one contract \((y^{IB}, s^{IB})\) to the firm.

The contracts under these cases of ignorance are discussed in Appendix D. Note that those contracts are not optimal in the original problem with information asymmetry. Next, we analyze how the ignorance affects the insurer and the firm.

### 6.3. Impacts of Ignorance

First, we characterize how each type of ignorance affects the insurer’s profit. It is intuitive that the insurer cannot have a better profit when it erroneously ignores the information asymmetry. We now formally show this statement and describe the profit loss due to the ignorance based on the firm’s purchasing decision.

**Proposition 14.** The firm has the following purchasing decisions when the insurer ignores the information asymmetry:

(i) Given the contract \((y^{IA}, s^{IA})\), the high-demand firm will not purchase insurance, and the low-demand firm will purchase insurance.

(ii) Given the contracts \((y^{IM}_H, s^{IM}_H), (y^{IM}_L, s^{IM}_L)\), the high-demand firm will not purchase insurance. The low-demand firm will not purchase the contract \((y^{IM}_L, s^{IM}_L)\), but may choose the contract \((y^{IM}_H, s^{IM}_H)\) instead.
Given the contract \((y^H, s^H)\), the high-demand firm will not purchase insurance, and the low-demand firm may purchase insurance.

When the insurer ignores information asymmetry, the constraint (IR-H) would be violated, and thus, the high-demand firm will not purchase the insurance. This is because the optimal contract \((y^H, s^H)\) already exploits all of the surplus of this firm. Therefore, the aforementioned types of ignorance will make the high-demand firm unwilling to purchase the insurance. On the other hand, the low-demand firm has positive net benefit from the contract \((y^L, s^L)\), but it does not necessarily benefit from the other contracts. Especially, if the insurer ignores moral hazard, the contracts fail to meet the constraint (IC-LH); therefore the low-demand firm may have incentive to purchase the high-demand contract \((y^L, s^L)\) if it is profitable, and never purchase the low-demand contract \((y^L, s^L)\).

We let \(R_H(y_H, s_H)\) be the insurer’s profit from the high-demand firm, and \(R_L(y_L, s_L)\) be the profit from the low-demand firm, where \(R(y_H, s_H, y_L, s_L) = R_H(y_H, s_H) + R_L(y_L, s_L)\), and

\[
R_H(y_H, s_H) = \alpha \left\{ y_H - \rho \left( \beta s_H p E_c \left[ \min (D_{HH}, K) - \min (D_{HH}, e_{HH}^*(s_H)K) \right] \right) \\
+ (1 - \beta) s_H p E_c \left[ \min (D_{HL}, K) - \min (D_{HL}, e_{HL}^*(s_H)K) \right] \right\},
\]

\[
R_L(y_L, s_L) = (1 - \alpha) \left\{ y_L - \rho \left( \beta s_L p E_e \left[ \min (D_{LH}, K) - \min (D_{LH}, e_{LH}^*(s_L)K) \right] \right) \\
+ (1 - \beta) s_L p E_e \left[ \min (D_{LL}, K) - \min (D_{LL}, e_{LL}^*(s_L)K) \right] \right\}.
\]

Therefore, the actual profits the aforementioned cases of ignorance are

\[
R(y^A, s^A, y^A, s^A) = R_L(y^L, s^L), \quad (6.1)
\]

\[
R(y^M, s^M, y^M, s^M) = \begin{cases} 
R_L(y^M, s^M) & \text{if } E_{\delta, Z}[\pi^L(K, y^M, s^M)] > E_{\delta, Z}[\pi^N(K)], \\
0 & \text{otherwise},
\end{cases} \quad (6.2)
\]

\[
R(y^B, s^B, y^B, s^B) = \begin{cases} 
R_L(y^B, s^B) & \text{if } E_{\delta, Z}[\pi^L(K, y^B, s^B)] > E_{\delta, Z}[\pi^N(K)], \\
0 & \text{otherwise}.
\end{cases} \quad (6.3)
\]

Moreover, we let \(L^A, L^M, L^B\) denote the insurer’s profit loss when ignoring adverse selection, moral hazard and both, respectively. Then,

\[
L^A = R(y^*_H, s^*_H, y^*_L, s^*_L) - R(y^A, s^A, y^A, s^A), \quad (6.4)
\]

\[
L^M = R(y^*_H, s^*_H, y^*_L, s^*_L) - R(y^M, s^M, y^M, s^M), \quad (6.5)
\]

\[
L^B = R(y^*_H, s^*_H, y^*_L, s^*_L) - R(y^B, s^B, y^B, s^B). \quad (6.6)
\]
The following proposition provides a lower bound of the profit loss when the insurer ignores information asymmetry.

**Proposition 15.** Given the optimal contract for the high-demand firm \((y^*_H, s^*_H)\), there exists a lower bound \(L\) of the profit loss under any type of ignorance. That is, \(\min\{L^{IA}, L^{IM}, L^{IB}\} \geq L\), where

\[
L = R_H(y^*_H, s^*_H) - (1 - \alpha)(1 + \lambda)s^*_H\gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)] \\
= \alpha \left\{ y^*_H - \rho \left( \beta s^*_H p E_\epsilon \left[ \min (D_{HH}, K) - \min (D_{HH}, e_{HH}(s^*_H)K) \right] \right) \right\} + (1 - \beta) s^*_H p E_\epsilon \left[ \min (D_{HL}, K) - \min (D_{HL}, e_{HL}(s^*_H)K) \right] \\
+ (1 - \alpha)(1 + \lambda)s^*_H\gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)] \geq 0.
\] (6.9)

Recall that, based on Proposition 14, the insurer always loses the profit from the high-demand firm when the insurer ignores any type of information asymmetry. Although the insurer may get extra profit from the low-demand firm, Proposition 15 shows that the loss from the high-demand firm is always greater than or equal to any additional profit from the low-demand firm. Thus, there always exists a cost for the insurer when it ignores any type of information asymmetry. However, which type of ignorance has the highest cost depends on the problem parameters. We provide numerical examples to show the cost of ignorance under different parameters in Section 7.2.

The ignorance of information asymmetry may also influence the firm’s profit. Under the optimal contracts, the high-demand firm’s profits with and without BI insurance are the same because the constraint (IR-H) is binding (as shown in Proposition 5). Under other contracts that ignore information asymmetry, the high-demand firm will not purchase any insurance, and hence, its profit would be equal to the profit without BI insurance. Thus, its profit does not change when the insurer ignores information asymmetry. However, the low-demand firm’s profit is affected by the insurer’s ignorance. Whether the low-demand firm benefits from the ignorance depends on the parameters, and we provide a detailed numerical analysis in Section 7.2.

We are also interested in how the insurance contracts change when the insurer ignores information asymmetry, and how those changes affect the firm’s recovery effort. We note that if the optimal coverage percentage is a corner solution (i.e., 0 or \(\bar{s}\)), then the contract under ignorance may be insensitive to any type of ignorance. Thus, as in Section 6.1, we consider the case in which the insurance contracts are sensitive to ignoring information asymmetry (i.e., the optimal coverage percentage is a stationary point of the profit function). Based on this, we derive the following properties.
PROPOSITION 16. With regard to insurance coverages under the different contracts, we have:

(i) \( s_{LM}^L \geq s_{L}^* \geq s_{IA} \geq s_{HM} \).
(ii) \( s_{LM}^L \geq s_{LB} \geq \max\{s_{IA}, s_{HM}^L\} \).

A simple verbal paraphrase of Proposition 16 is as follows: if the insurer ignores adverse selection, then it offers a single contract to the firm, where the coverage percentage is higher than that of the high-demand contract without ignorance but lower than that of the low-demand contract without ignorance. If the insurer ignores moral hazard, then it offers a contract to the low-demand firm, where the coverage percentage is higher relative to the low-demand contract in the absence of ignorance. Thus, ignoring adverse selection and moral hazard has distinct effects on the coverage of the low-demand contract.

To explain the results in Proposition 16, we note that, when the insurer ignores adverse selection, it erroneously assumes the firm’s early demand forecast is the average demand forecast \( \bar{\mu} \). Thus, in this case, the insurer would choose the insurance coverage between those coverages in the optimal contracts without ignorance. When the insurer ignores moral hazard, it assumes that the firm’s optimal effort is perfectly predictable. Thus, the insurer would choose a higher insurance coverage to exploit the firm’s surplus based on the predicted recovery effort.

According to Proposition 14, the contract \((y_{LM}^L, s_{LM}^L)\) would never be purchased by the firm with the low demand forecast. Thus, we focus on the other three contracts under ignorance to observe their impact on the low-demand firm.

PROPOSITION 17. Under any type of ignorance, if the low-demand firm purchases insurance, then the premium of the contract under ignoring both adverse selection and moral hazard is the highest among all types of ignorance. That is, \( y_{LB}^I \geq \max\{y_{IA}, y_{HM}^L\} \).

According to Proposition 17, although the low-demand firm obtains the highest coverage percentage under the contract that ignores both adverse selection and moral hazard, it also has to pay the highest premium in this case. As a result, if the premium is sufficiently high, then the low-demand firm would not purchase this contract (as in Proposition 14).

Finally, we characterize how the firm’s recovery effort changes under ignorance.

PROPOSITION 18. Under any type of ignorance, the high-demand firm always increases its recovery effort, and the low-demand firm increases its recovery effort when it does not purchase insurance. If the low-demand firm purchases insurance, we have:

(i) \( e_{L}^*(s_{L}) \leq e_{L}^*(s_{IA}) \)
(ii) \( e_{L}^*(s_{LB}) \leq \min\{e_{L}^*(s_{IA}), e_{L}^*(s_{HM})\} \)
If the insurer ignores adverse selection, the low-demand firm would put more effort into recovery relative to no ignorance. In addition, its recovery effort under ignoring both is less than the effort under ignoring only one type of information asymmetry if the firm purchases insurance. It is perhaps interesting that the impacts of ignoring adverse selection on this firm’s recovery effort may be different between the cases with and without ignoring moral hazard. This is because, when the insurer only ignores moral hazard, the low-demand firm may choose the contract designed for the high-demand firm, which has a much lower coverage relative to the contract under ignoring both.

7. Numerical Results
This section provides illustrative numerical results to support the analytical findings in the previous sections. To that end, we first investigate the optimal contracts and the profit of the firm and the insurer in Section 7.1, and then we analyze the impact of ignorance in Section 7.2.

7.1. A Case Study for the Optimal Contracts
To examine a realistic setting, let us consider the 2011 Thailand floods, which serves a motivating example for our work (see Adachi et al. 2016 for empirical evidence of moral hazard in the BI insurance practice in this example). Toyota’s manufacturing was significantly affected by these floods, suffering a substantial loss due to these floods (Haraguchi and Lall 2015), and accordingly, we consider Toyota Motor Thailand Corporation (TMT) to calibrate our numerical example. First, TMT’s current production is 760,000 units/year (Toyota Motor Thailand Co., Ltd. 2017); thus, we let \( K = 760 \) (where quantities are measured in thousands). According to Marklines (2017) and Temphairojana et al. (2017), TMT’s annual domestic and export sales in 2016 were approximately 244,317 and 316,854 units, respectively, and thus, its total annual sales were roughly 561,171 units in 2016. Moreover, Kyoichi Tanada, the president of TMT, stated “2016 was a tough year” (Temphairojana et al. 2017). Therefore, we deduce that TMT’s demand in 2016 corresponds to a low market condition; consequently, we let \( \mu_L = 560 \). We also set the high demand forecast as \( \mu_H = 660 \), which is between the capacity and the low demand forecast, and for simplicity, we use a uniform distribution for the early demand forecast. That is, \( \alpha = 0.5 \), and the mean of early demand forecast \( \bar{\mu} = 610 \).

With regard to the forecast update, we consider a uniform distribution with relatively smaller quantities; we assume that \( \beta = 0.5, \delta_H = 25 \) and \( \delta_L = -25 \), so that \( \delta \) has a lower variation than that of \( \mu \). Furthermore, Toyota’s order fulfillment rate is 96%-98% in average (Ludwig 2015). With 98% fill rate, we can estimate the parameter of the logistic demand shocks as \( \gamma = -(760 - 610)/\ln(1 - 0.98) = 38.34 \approx 40 \) based on (3.4). Next, we note that the abnormally large rainfall that causes
floods in Thailand is estimated to be once in 50 years (METI Japan 2012), and therefore we let the probability of disruptions $\rho = 1/50 = 0.02$. According to Toyota Motor Co., Ltd. (2016), the unit operating profit for a vehicle is $5280$ in average; we let $p = 5.28$ and suppose that the unit cost of recovery is $c = 3$. Finally, Dong and Tomlin (2012) consider the unit disruption penalty cost to be within 1 to 5, and thus, we let $\lambda = 1$ in our numerical studies.

Case 1 in Table 2 corresponds to the setting described above, which serves a benchmark of our numerical studies. We check for the robustness of our numerical results in Cases 2-8, which are also displayed in Table 2. Table 3 shows the optimal contracts and the optimal profit of the insurer, and Table 4 shows the firm’s profit under those optimal contracts.

$$\begin{array}{cccccccccccc}
\text{Case} & \alpha & \beta & K & \mu_H & \mu_L & \delta_H & \delta_L & \gamma & \rho & \lambda & \mu & c \\
1 & 0.5 & 0.5 & 760 & 660 & 560 & 25 & -25 & 40 & 0.02 & 1 & 5.28 & 3 \\
2 & 0.6 & 0.5 & 760 & 660 & 560 & 25 & -25 & 40 & 0.02 & 1 & 5.28 & 3 \\
3 & 0.4 & 0.5 & 760 & 660 & 560 & 25 & -25 & 40 & 0.02 & 1 & 5.28 & 3 \\
4 & 0.5 & 0.5 & 760 & 610 & 560 & 25 & -25 & 40 & 0.02 & 1 & 5.28 & 3 \\
5 & 0.5 & 0.5 & 760 & 710 & 560 & 25 & -25 & 40 & 0.02 & 1 & 5.28 & 3 \\
6 & 0.5 & 0.6 & 760 & 660 & 560 & 20 & -30 & 40 & 0.02 & 1 & 5.28 & 3 \\
7 & 0.5 & 0.4 & 760 & 660 & 560 & 30 & -20 & 40 & 0.02 & 1 & 5.28 & 3 \\
8 & 0.5 & 0.5 & 760 & 660 & 560 & 25 & -25 & 40 & 0.02 & 1 & 5.28 & 2 \\
\end{array}$$

Table 2 The Parameters of the Numerical Results

$$\begin{array}{cccccccc}
\text{Case} & y_{H}^* & s_{H}^* & y_{L}^* & s_{L}^* & R(y_{H}^*, s_{H}^*, y_{L}^*, s_{L}^*) \\
1 & 2.234 & 26.416\% & 2.535 & 28.751\% & 0.646 \\
2 & 2.305 & 27.020\% & 2.531 & 28.751\% & 0.655 \\
3 & 2.115 & 25.386\% & 2.542 & 28.751\% & 0.639 \\
4 & 2.615 & 28.284\% & 2.677 & 28.751\% & 0.759 \\
5 & 0 & 0\% & 2.724 & 28.751\% & 0.397 \\
6 & 2.242 & 26.461\% & 2.538 & 28.752\% & 0.649 \\
7 & 2.231 & 26.404\% & 2.534 & 28.751\% & 0.646 \\
8 & 1.318 & 30.475\% & 1.822 & 37.508\% & 0.391 \\
\end{array}$$

Table 3 The Optimal Contracts and Profit of the Insurer

According to the numerical results, we make the following observations. First, the high-demand firm does not benefit from the insurance in any of the cases, because the insurer exploits all of its surplus from the BI insurance. On the other hand, the low-demand firm benefits from the insurance if the insurer offers both contracts to the firm, and this benefit is due to the information asymmetry. In Case 5, the insurer does not offer a contract to the high-demand firm because such an offer would not be profitable to the insurer when $\mu_H$ is sufficiently high, which corresponds
Table 4 The Firm’s Profit under the Optimal Contracts

to the complement of the conditions in Proposition 7. In this case, the low-demand firm cannot benefit from the insurance because there is no other contract that the firm can choose, and the insurer then can exploit all of the low-demand firm’s surplus.

7.2. Cost of Ignorance

Next, we compare the cost of ignorance among different types of ignorance and under different parameter values. For Cases 1-8 displayed in Table 2, we show in Table 5 the percentage of the insurer’s profit loss due to ignorance, which is given by

\[
\text{Percentage of Profit Loss} = \frac{\text{Optimal Profit} - \text{Actual Profit with Ignorance}}{\text{Optimal Profit}}.
\]

There are some surprising results in Table 5. First, among all these 8 cases, the cost of ignorance is not necessarily the greatest when ignoring both types of information asymmetry, which might appear counter-intuitive. This is because the contracts designed with ignorance neglect key details of the actual problem. According to Proposition 14, under ignorance, the insurer always loses the profit from the high-demand firm. When the high-demand firm is left out of the problem, the contracts that ignore both adverse selection and moral hazard may end up better targeting the
low-demand firm, earning more profit from that firm; thus, the profit loss could be the lowest. Second, we find that the cost of ignoring moral hazard seems to be higher than that when ignoring adverse selection. However, these two statement are not always true. We next provide another numerical study to show the different results.

To demonstrate a broader range of numerical results, we also consider the additional cases in Table 6, and we show in Table 7 the corresponding cost of ignorance.

First, although ignoring both adverse selection and moral hazard may better target the firm with the lower demand forecast, such ignorance may yield no purchases by the low-demand firm (as in Proposition 14). In such cases, the insurer earns no profit, and the profit loss is 100% (see Cases 11-16 in Table 7), which is the greatest among all types of ignorance. A similar result may also happen under ignoring moral hazard, leading to 100% profit loss (see Case 12 in Table 7). Moreover, according to Table 7, whether ignoring adverse selection costs more than ignoring moral hazard depends on the problem parameters. As a result, it is not always the case that ignoring
moral hazard has higher cost than ignoring adverse selection. For example, consider the effect of \(\alpha\). In Figure 5(a), when \(\alpha\) is low, the insurer’s cost of ignoring moral hazard or ignoring both types of information asymmetry is 100%. However, this cost suddenly drops when \(\alpha\) becomes large (and the low-demand firm becomes willing to purchase the contract). In this example, the cost of ignoring adverse selection is the greatest when \(\alpha\) is sufficiently large. This numerical example also shows that each type of ignorance could make the insurer the highest cost under different parameters.

![Graph showing the impact of different contracts](image)

**Figure 5** The Impact of Different Contracts. The problem parameters are \(K = 200, \mu_H = 160, \mu_L = 120, \beta = 0.5, \delta_H = 25, \delta_L = -25, \rho = 0.05, \gamma = 15, p = 10, c = 4, \) and \(\lambda = 1.\)

With regard to the firm’s profit under different contracts, recall that the high-demand firm has same profit because it makes no purchase. Thus, we focus on the low-demand firm’s profit. Figure 5(b) shows that, when \(\alpha\) is low, the low-demand firm has less profit under the optimal contract than under the other contracts. However, once \(\alpha\) is large enough, the low-demand firm can benefit the most from the optimal contracts. It is because the insurer has a lower probability of earning profit from the low-demand firm so it forgoes more profit to the low-demand firm with less cost, and accordingly, it can earn more from the high-demand firm. When the insurer ignores adverse selection, the firm observing the low demand forecast benefits more when \(\alpha\) increases because the contract is closer to the contract for the high-demand firm, and thus, this contract is more beneficial to the low-demand firm.

8. Conclusion

In this paper, we study the impact of adverse selection and moral hazard on optimal insurance contracts with operational considerations. We find that the compensation for a firm’s revenue loss due to a disruption is truncated by the firm’s capacity. The low-demand firm (i.e., the firm with a low demand forecast) is less affected by the capacity constraint and thus has higher net profit
from BI insurance relative to the high-demand firm. Accordingly, the low-demand firm would be offered a contract with higher coverage and premium.

We also investigate how optimal insurance contracts are affected by the firm’s operational and informational characteristics, and study the impact of ignoring information asymmetry. The insurer always loses profit from the high-demand firm (i.e., the firm with a high demand forecast) when ignoring information asymmetry, but it does not necessarily incur the greatest loss when ignoring both adverse selection and moral hazard. The high-demand firm’s profit with BI insurance is always equal to the profit without insurance; thus, it is not affected by any ignorance. The low-demand firm’s profit is affected by ignorance, but it does not necessarily benefits from the insurer’s ignorance.

Our work can be extended by two directions. First, in our paper, we consider a single-period problem for an insurer and a firm, but BI insurance can be effective over a longer planning horizon. Information asymmetry may have different impact in a long-term planning problem. Second, a disruption may not only affect the firm but also its upstream suppliers and downstream firms in a supply chain. One can investigate how BI insurance should be tailored to different business units in a supply chain and how BI insurance mitigates the income loss caused by a supply chain disruption.

References


Appendix A: Optimal Effort with BI Insurance

Given $D_{ij}$, the firm’s expected profit with BI insurance is

$$\pi_{ij}^e(K, e, y, s) = (1 + \lambda) \left\{ -y - ceK + pE_e \left[ \min (D_{ij}, eK) \right] + spE_e \left[ \min (D_{ij}, K) - \min (D_{ij}, eK) \right] \right\}$$

$$= (1 + \lambda) \left\{ -y - ceK + p \int_0^{eK} x f_{ij}(x) dx + \int_{eK}^\infty eK f_{ij}(x) dx \right\} + sp \left[ \int_{eK}^K (x - eK) f_{ij}(x) dx + \int_{eK}^\infty (K - eK) f_{ij}(x) dx \right] - \lambda p E_e \left[ \min (D_{ij}, K) \right].$$

Based on this, the (unconstrained) first-order optimality condition is

$$\frac{d}{de} \pi_{ij}^e(K, e, y, s) = (1 + \lambda) \left\{ -cK + (pK - spK) \left[ 1 - F_{ij}(eK) \right] \right\} = 0.$$

The second-order derivative of the profit function with respect to effort $e$ is negative. Therefore, the firm’s profit is a concave function of effort. Given that $s < \frac{p - c}{p}$, the effort $e$ satisfying the (unconstrained) first-order optimality condition is $\frac{1}{K} F_{ij}^{-1} \left( \frac{p(1-s) - c}{p(1-s)} \right)$. However, this value may violate the constraint that $e \leq 1$. Thus, the optimal effort subject to said constraint is

$$e_{ij}^e(s) = \begin{cases} \frac{1}{K} F_{ij}^{-1} \left( \frac{p(1-s) - c}{p(1-s)} \right) = \frac{1}{K} \left[ \mu_i + \delta_j + \gamma \ln \left( \frac{p(1-s) - c}{e} \right) \right] & \text{if } F_{ij}^{-1} \left( \frac{p(1-s) - c}{p(1-s)} \right) < K, \\ 1 & \text{otherwise.} \end{cases}$$

Appendix B: Proof of the Propositions

**Proof of Proposition 1.** Let us take the first and second-order derivative of the firm’s optimal effort $e_{ij}^e(s)$ with respect to $s$:

$$\frac{d}{ds} e_{ij}^e(s) = \begin{cases} \frac{-p \gamma}{K(p(1-s) - c)} < 0 & \text{if } e_{ij}^e(s) = \frac{1}{K} F_{ij}^{-1} \left( \frac{p(1-s) - c}{p(1-s)} \right), \\ 0 & \text{if } e_{ij}^e(s) = 1, \end{cases}$$

$$\frac{d^2}{ds^2} e_{ij}^e(s) = \begin{cases} \frac{-p^2 \gamma}{[K(p(1-s) - c)]^2} < 0 & \text{if } e_{ij}^e(s) = \frac{1}{K} F_{ij}^{-1} \left( \frac{p(1-s) - c}{p(1-s)} \right), \\ 0 & \text{if } e_{ij}^e(s) = 1. \end{cases}$$

Since the first-order derivative and the second-order derivative are both non-positive, $e_{ij}^e(s)$ is non-increasing and concave in $s$. Q.E.D.

**Proof of Proposition 2.** We take the first- and second-order derivatives of the firm’s expected profit with respect to $s$. First, if $e_{ij}^e(s) = 1$, this implies that the resumed capacity is exactly the
initial capacity $K$. Therefore, no revenue loss can be claimed and the firm receives no compensation. As a result, the coverage percentage $s$ does not influence the expected profit $\pi_{ij}^f(K, y, s)$ in this case. For $j \in \{H, L\}$, if $e_{ij}^*(s) = 1$, then $\frac{d}{ds}\pi_{ij}^f(K, y, s) = 0$.

On the other hand, if $e_{ij}^*(s) \neq 1$, the first-order derivative of $E_{\delta, Z}[\pi_i^f(K, y, s)]$ with respect to $s$ is

$$\frac{d}{ds}E_{\delta, Z}[\pi_i^f(K, y, s)] = \beta \rho \frac{d}{ds}\pi_{iH1}^f(K, y, s) + \beta \rho \frac{d}{ds}\pi_{iL1}^f(K, y, s)$$

$$= \beta \rho (1 + \lambda) \left\{ \int_0^K [x - e_{iH}^*(s)K] f_{iH}(x) dx + \int_0^K [K - e_{iH}^*(s)K] f_{iH}(x) dx \right\}$$

$$+ (1 - \beta) \rho (1 + \lambda) \left\{ \int_{e_{iL}^*(s)K}^K [x - e_{iL}^*(s)K] f_{iL}(x) dx + \int_0^K [K - e_{iL}^*(s)K] f_{iL}(x) dx \right\}$$

$$= \rho (1 + \lambda) \left\{ K - \gamma (\beta w(\mu_i, \delta_H) + (1 - \beta) w(\mu_i, \delta_L)) + \gamma \ln \left( \frac{p(1 - s)}{\rho (1 - s) - c} \right) \right\} \geq 0.$$ 

Thus, in general, the first-order derivative is non-negative on $s$. For the second-order derivative, we note that, if $e_{ij}^*(s) = 1$, then $\frac{d^2}{ds^2}\pi_{ij}^f(K, y, s) = 0$. If $e_{ij}^*(s) \neq 1$, then

$$\frac{d^2}{ds^2}E_{\delta, Z}[\pi_i^f(K, y, s)] = \beta \rho \frac{d^2}{ds^2}\pi_{iH1}^f(K, y, s) + (1 - \beta) \rho \frac{d^2}{ds^2}\pi_{iL1}^f(K, y, s)$$

$$= \beta \rho (1 + \lambda) \frac{p c \gamma}{(1 - s) (p (1 - s) - c)} + (1 - \beta) \rho (1 + \lambda) \frac{p c \gamma}{(1 - s) (p (1 - s) - c)} \geq 0.$$ 

Therefore, in general, the second-order derivative is non-negative as well, and the expected profit is non-decreasing and convex in $s$. Q.E.D.

**Proof of Proposition 3.** To obtain the desired result, we will take the derivative of the firm’s expected profit with respect to $s$ and $\mu_i$. Based on the proof in Proposition 2, we deduce that, if $e_{ij}^*(s) = 1$ for any $j \in \{H, L\}$, then $\frac{d^2}{ds^2}\pi_{ij}^f(K, y, s) = 0$ because $\frac{d}{ds}\pi_{ij}^f(K, y, s) = 0$. If $e_{ij}^*(s) \neq 1$, we have

$$\frac{d^2}{ds d\mu_i}E_{\delta, Z}[\pi_i^f(K, y, s)] = \beta \rho \frac{d^2}{ds d\mu_i}\pi_{iH1}^f(K, y, s) + (1 - \beta) \rho \frac{d^2}{ds d\mu_i}\pi_{iL1}^f(K, y, s)$$

$$= - \beta \rho (1 + \lambda) p \frac{\exp \left( \mu_i^* + \delta_H \frac{\rho}{\gamma} \right)}{\exp \left( \frac{\rho}{\gamma} \right) + \exp \left( \mu_i^* + \delta_L \frac{\rho}{\gamma} \right)} - (1 - \beta) \rho (1 + \lambda) p \frac{\exp \left( \mu_i^* + \delta_L \frac{\rho}{\gamma} \right)}{\exp \left( \frac{\rho}{\gamma} \right) + \exp \left( \mu_i^* + \delta_H \frac{\rho}{\gamma} \right)} < 0.$$ 

Thus, the marginal profit per coverage percentage $s$ is non-increasing in $\mu_i$. Q.E.D.
Proof of Proposition 4. Let us first take the first-order derivative of the firm’s net profit from BI insurance with respect to $\mu_i$:

$$
\frac{d}{d\mu_i} \left\{ E_{\delta,Z} [\pi_i^I(K,y,s)] - E_{\delta,Z} [\pi_i^N(K)] \right\} = \beta \rho \frac{d}{d\mu_i} \left\{ \pi_{iH1}(K,y,s) - \pi_{iH1}^N(K) \right\} + (1 - \beta) \rho \frac{d}{d\mu_i} \left\{ \pi_{iL1}(K,y,s) - \pi_{iL1}^N(K) \right\}.
$$

For any $j \in \{H, L\}$, because the optimal effort $\hat{e}_{ij}$ and $e_{ij}^*(s)$ may be 1, we consider three different cases:

1. For $j \in \{H, L\}$, $\hat{e}_{ij} < 1$

   In this case, both $\hat{e}_{ij}$ and $e_{ij}^*(s)$ are less than 1, and thus we have

   $$
   \frac{d}{d\mu_i} \left\{ \pi_{ij1}(K,y,s) - \pi_{ij1}^N(K) \right\} = -ps \frac{\exp \left( \frac{\mu_i + \delta_j}{\gamma} \right)}{\exp \left( \frac{K}{\gamma} \right) + \exp \left( \frac{\mu_i + \delta_j}{\gamma} \right)} < 0.
   $$

2. For $j \in \{H, L\}$, $\hat{e}_{ij} = 1$ and $e_{ij}^*(s) < 1$

   In this case, the firm would resume its entire capacity without BI insurance, but it would resume its capacity partially with BI insurance. Hence,

   $$
   \frac{d}{d\mu_i} \left\{ \pi_{ij1}(K,y,s) - \pi_{ij1}^N(K) \right\} = p(1 - s) \frac{\exp \left( \frac{\mu_i + \delta_j}{\gamma} \right)}{\exp \left( \frac{K}{\gamma} \right) + \exp \left( \frac{\mu_i + \delta_j}{\gamma} \right)} - c.
   $$

   Since $e_{ij}^*(s) < 1$ implies that $K > \mu_i + \delta_j + \gamma \ln \left( \frac{p(1-s)-c}{c} \right)$, thus we can find out that

   $$
   \frac{d}{d\mu_i} \left\{ \pi_{ij1}(K,y,s) - \pi_{ij1}^N(K) \right\} < p(1 - s) \frac{\exp \left( \frac{\mu_i + \delta_j}{\gamma} \right)}{\exp \left( \frac{\mu_i + \delta_j + \gamma \ln \left( \frac{p(1-s)-c}{c} \right)}{\gamma} \right) + \exp \left( \frac{\mu_i + \delta_j}{\gamma} \right)} - c = 0.
   $$

3. For $j \in \{H, L\}$, $\hat{e}_{ij} = 1$ and $e_{ij}^*(s) = 1$

   In this case, the value of $\pi_{ij1}(K,y,s)$ and $\pi_{ij1}^N(K)$ are the same, and thus,

   $$
   \pi_{ij1}(K,y,s) - \pi_{ij1}^N(K) = 0.
   $$

In all cases, the derivative with respect to $\mu_i$ is non-positive. We then can conclude that given a insurance contract $(y,s)$, the firm’s net profit from BI insurance is non-increasing in the early demand forecast $\mu_i$. Q.E.D.

Proof of Proposition 5. First, suppose the constraint (IR-II) is not binding under the optimal contracts, then we have

$$
E_{\delta,Z} [\pi^I_H(K,y^*_H,s^*_H)] > E_{\delta,Z} [\pi^N_H(K)].
$$
Based on the constraint (IC-LH) and Proposition 4, we have

\[
E_{\delta,Z}[\pi_L^I(K, y_H^*, s_H^*)] - E_{\delta,Z}[\pi_L^N(K)] \geq E_{\delta,Z}[\pi_L^I(K, y_H^*, s_H^*)] - E_{\delta,Z}[\pi_L^N(K)] \\
\geq E_{\delta,Z}[\pi_H^I(K, y_H^*, s_H^*)] - E_{\delta,Z}[\pi_H^N(K)] > 0.
\]

Therefore, the insurer’s profit can be improved by increasing \(y_H^*\) and \(y_L^*\) by a small number \(\varepsilon > 0\) (without violating any constraints), which contradicts the fact that \((y_H^*, s_H^*)\) and \((y_L^*, s_L^*)\) are the optimal contracts. As a result, the constraint (IR-H) is binding under the optimal contracts.

Given that the constraint (IR-H) is binding under the optimal contracts, suppose now that (IC-LH) is not binding under the optimal contracts. Then,

\[
E_{\delta,Z}[\pi_L^I(K, y_L^*, s_L^*)] - E_{\delta,Z}[\pi_L^N(K)] > E_{\delta,Z}[\pi_L^I(K, y_H^*, s_H^*)] - E_{\delta,Z}[\pi_L^N(K)] \\
\geq E_{\delta,Z}[\pi_H^I(K, y_H^*, s_H^*)] - E_{\delta,Z}[\pi_H^N(K)] = 0.
\]

Thus, the insurer’s profit can also be improved by increasing \(y_L^*\) by a small number \(\varepsilon > 0\) (without violating any constraints). This contradicts the fact that \((y_H^*, s_H^*)\) and \((y_L^*, s_L^*)\) are the optimal contracts. As a result, the constraint (IC-LH) is binding under the optimal contracts.

Because the constraints (IC-LH) and (IR-H) are binding, we deduce the following for the constraint (IR-L):

\[
E_{\delta,Z}[\pi_L^I(K, y_L^*, s_L^*)] - E_{\delta,Z}[\pi_L^N(K)] = E_{\delta,Z}[\pi_L^I(K, y_H^*, s_H^*)] - E_{\delta,Z}[\pi_L^N(K)] \\
\geq E_{\delta,Z}[\pi_H^I(K, y_H^*, s_H^*)] - E_{\delta,Z}[\pi_H^N(K)] = 0.
\]

Therefore, the constraint (IR-L) is redundant when the constraints (IC-LH) and (IR-H) are binding.

Lastly, because the constraints (IC-LH) and (IR-H) are binding under the optimal contracts, we have the following for the constraint (IC-HL): \(E_{\delta,Z}[\pi_H^I(K, y_H^*, s_H^*)] \geq E_{\delta,Z}[\pi_H^I(K, y_L^*, s_L^*)]\), which implies that

\[
E_{\delta,Z}[\pi_H^I(K, y_H^*, s_H^*)] - E_{\delta,Z}[\pi_H^N(K)] \geq E_{\delta,Z}[\pi_H^I(K, y_L^*, s_L^*)] - E_{\delta,Z}[\pi_H^N(K)].
\]

If the constraint (IC-HL) is not satisfied, then fixed \((y_H^*, s_H^*)\), \(y_L^*\) should be lower or \(s_L^*\) should be higher to satisfy the constraint (IC-HL), which contradicts the fact that (IC-LH) is binding; if \(y_H^*\) and \(y_L^*\) simultaneously decrease or \(s_H^*\) and \(s_L^*\) simultaneously increase to satisfy the constraints (IC-HL) and (IC-LH), then this contradicts the fact that (IR-H) is binding. Thus, the constraint (IC-HL) is redundant when the constraints (IC-LH) and (IR-H) are binding under the optimal contracts. Q.E.D.
Proofs of Propositions 6 and 7. To show the properties of the insurer’s profit function $R(s_H, s_L)$, we first compute the first-order derivatives and Hessian matrix with respect to $s_H$ and $s_L$. Given that $K > \mu_H + \delta_H + \gamma \ln \left( \frac{\hat{c}_{\hat{s}_L}}{c} \right) > \mu_L + \delta_H + \gamma \ln \left( \frac{\hat{c}_{\hat{s}_L}}{c} \right)$, we have

$$\frac{d}{ds_H} R(s_H, s_L) = \rho \left\{ (1 - \alpha)(1 + \lambda) \gamma E_{\delta} \left[ w(\mu_L, \delta) - w(\mu_H, \delta) \right] - \alpha \frac{c \gamma s_H}{(p(1-s_H)-c)(1-s_H)} \right\} + \alpha \lambda \left( K + \gamma \ln \left[ \frac{p(1-s_H)}{p(1-s_L)-c} \right] - \gamma E_{\delta} \left[ w(\mu_H, \delta) \right] \right), \quad (B.1)$$

$$\frac{d^2}{ds_H^2} R(s_H, s_L) = \alpha \rho \frac{\lambda \gamma \gamma E_{\delta} \left[ w(\mu_L, \delta) \right]}{(p(1-s_H)-c)(1-s_H)^2}, \quad (B.2)$$

$$\frac{d^2}{ds_L^2} R(s_H, s_L) = (1 - \alpha) \rho \frac{\lambda \gamma \gamma E_{\delta} \left[ w(\mu_L, \delta) \right]}{(p(1-s_L)-c)(1-s_L)^2}, \quad (B.3)$$

$$\frac{d^2}{ds_H s_L} R(s_H, s_L) = 0. \quad (B.4)$$

Since there is no interaction term of $s_H$ and $s_L$ in the profit function, we can check the optimality separately. First, a sufficient condition for the concavity of $R(s_H, s_L)$ in terms of $s_L$ is:

$$\lambda(p(1-s_L) - c)(1-s_L) + c - p(1-s_L^2) \leq 0. \quad (B.5)$$

The left-hand side of the preceding inequality is a quadratic and convex function of $s_L$. Thus, the preceding sufficient condition is satisfied within the two roots of the quadratic function. Since $0 \leq s_L \leq \frac{\hat{c}_{\hat{s}_L}}{p}$, said quadratic function achieves its highest value either at $s_L = 0$ or $s_L = \frac{\hat{c}_{\hat{s}_L}}{p}$. Its value is $(\lambda - 1)(p - c)$ at $s_L = 0$, and $-\frac{(p-c)}{p}$ at $s_L = \frac{\hat{c}_{\hat{s}_L}}{p}$. As a result, we can conclude that, if $\lambda \leq 1$, then the profit function is concave in $s_L$. If $\lambda > 1$, the profit function is convex in $s_L$ when $s_L \leq \hat{s}_L$, and concave when $s_L \geq \hat{s}_L$, where $\hat{s}_L$ is the smaller root of that quadratic function, which is

$$\hat{s}_L = \frac{\lambda(2p - c) - \sqrt{4p(p-c) + (c\lambda)^2}}{2p(1+\lambda}).$$

Now, by the arguments in the proof in Proposition 2 and the condition that $K > \mu_L + \delta_H + \gamma \ln \left( \frac{\hat{c}_{\hat{s}_L}}{c} \right)$, we deduce the following for any $s_L$:

$$K + \gamma \ln \left[ \frac{p(1-s_L)}{p(1-s_L)-c} \right] - \gamma E_{\delta} \left[ w(\mu_H, \delta) \right] > 0.$$  

Therefore,

$$\frac{d}{ds_L} R(s_H, 0) = (1 - \alpha) \rho \left\{ \lambda \left( K + \gamma \ln \left[ \frac{p}{p-c} \right] - \gamma E_{\delta} \left[ w(\mu_L, \delta) \right] \right) \right\} > 0.$$
This implies that the profit function is increasing at $s_L = 0$. Therefore, if $\lambda > 1$, then the profit function is increasing and convex when $s_L \in [0, \hat{s}_L]$. In general, we conclude that the profit function is quasi-concave in $s_L$.

Similarly, we can check the optimality of the profit function on $s_H$. Due to the similar structure of the second-order derivative, the sufficient conditions for concavity are the same: if $\lambda \leq 1$, the profit function is concave in $s_H$, whereas if $\lambda > 1$, the profit function is convex in $s_H$ when $s_H \leq \hat{s}_H$, and concave when $s_H \geq \hat{s}_H$, where

$$
\hat{s}_H = \frac{\lambda(2p-c) - \sqrt{4p(p-c) + (c\lambda)^2}}{2p(1 + \lambda)}.
$$

In the first-order derivative of the profit function with respect to $s_H$, namely $\frac{d}{ds_H}R(s_H, s_L)$, there is an additional term $(1 - \alpha)(1 + \lambda)\gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)]$, which is a negative constant. Following the proof argument for quasi-concavity in $s_L$, we deduce that the profit function is increasing and convex in $0 \leq s_H \leq \hat{s}_H$ if $\frac{d}{ds_H}R(0, s_L)$ is non-negative. As a result, the following condition is sufficient for the quasi-concavity of profit function in $s_H$:

$$
pp\left\{(1 - \alpha)(1 + \lambda)\gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)] + \alpha \lambda \left( K + \gamma \ln \left[ \frac{p}{p-c} \right] - \gamma E_\delta [w(\mu_H, \delta)] \right) \right\} \geq 0. \quad \text{Q.E.D.}
$$

**Proof of Proposition 8.** We first compare $s_H'$ and $s_L'$. Note that, if $s_H' < s_L'$, then $s_H^* \leq s_L^*$, where $s_H'$ and $s_L'$ are as characterized in (5.6) and (5.7). Thus, we focus on the comparison between $s_H'$ and $s_L'$. We deduce from (5.7) that

$$
pp\left\{ \lambda \left( K + \gamma \ln \left[ \frac{p(1-s_L')}{p(1-s_L')-c} \right] - \gamma E_\delta [w(\mu_L, \delta)] \right) - \frac{c\gamma s_L'}{(p(1-s_L')-c)(1-s_L')} \right\} = 0.
$$

Because $\alpha \geq 0$ and $\mu_L \leq \mu_H$, this implies that

$$
pp\left\{ \alpha \lambda \left( K + \gamma \ln \left[ \frac{p(1-s_L')}{p(1-s_L')-c} \right] - \gamma E_\delta [w(\mu_H, \delta)] \right) - \alpha \frac{c\gamma s_L'}{(p(1-s_L')-c)(1-s_L')} \right\} < 0.
$$

Thus,

$$
pp\left\{ (1 - \alpha)(1 + \lambda)\gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)] - \alpha \frac{c\gamma s_L'}{(p(1-s_L')-c)(1-s_L')} \right\} < 0.
$$

Consequently, $\frac{d}{ds_H}R(s_L', s_L) < 0$; i.e., the first-order derivative of the profit function with respect to $s_H$ is negative at $s_H = s_L'$. Since the profit function is quasi-concave in $s_H$, this further implies that $s_H' < s_L'$. Therefore $s_H^* \leq s_L^*$. 
Next, we compare $y^*_H$ and $y^*_L$ via (5.3)-(5.4). We note that if $s^*_H = s^*_L$, then we have the premiums $y^*_H = g_H(s^*_H) = g_L(s^*_H, s^*_L) = y^*_L$ because all of the terms in (5.4) other than $g_H(s^*_H)$ would be 0. Now, consider the case where $s^*_H < s^*_L$. By the arguments used in the proof of Proposition 2, we have

$$
\frac{d}{ds} E_\delta \left\{ (p(1-s) - c) \left[ \mu_L + \delta + \gamma \ln \left( \frac{p(1-s) - c}{c} \right) \right] - p(1-s) \gamma \ln \left( \frac{p(1-s)}{c} \right) \right\} \geq 0.
$$

Thus, in this case, all of the terms in (5.4) other than $g_H(s^*_H)$ would be positive, and hence, $y^*_H = g_H(s^*_H) < g_L(s^*_H, s^*_L) = y^*_L$. As a result, we have $s^*_H \leq s^*_L$ and $y^*_H \leq y^*_L$. Q.E.D.

Proofs of Propositions 9-13. First, given that the profit function is quasi-concave and $s^*_H$ and $s^*_L$ satisfy the first-order optimality conditions, the profit function is concave at $s^*_H$ and $s^*_L$. That is, $\frac{d^2}{ds^2} R(s^*_H, s^*_L) \leq 0$ and $\frac{d^2}{ds^2} R(s^*_H, s^*_L) \leq 0$. Moreover, because there is no closed-form expression of $s^*_H$ and $s^*_L$, we use the implicit function theorem to compute the derivatives of $s^*_H$ and $s^*_L$ with respect to problem parameters. Based on this, we also compute the derivatives of $y^*_H$ and $y^*_L$ via (5.3) and (5.4).

Change in $\alpha$ (Proposition 9). First, note that

$$
\frac{d}{d\alpha} s^*_H = -\frac{\frac{d^2}{ds^2} R(s^*_H, s^*_L)}{\frac{d^2}{ds^2} R(s^*_H, s^*_L)}
$$

$$
= \frac{-1}{\frac{d^2}{ds^2} R(s^*_H, s^*_L)} \left\{ (1 + \lambda) \gamma E_\delta [w(\mu_H, \delta) - w(\mu_L, \delta)] - \frac{c \gamma s^*_H}{(p(1-s^*_H) - c)(1-s^*_H)} \right\}
$$

$$
+ \lambda \left( K + \gamma \ln \left( \frac{p(1-s^*_H)}{p(1-s^*_H) - c} \right) - \gamma E_\delta [w(\mu_H, \delta)] \right) \right\}
$$

In this derivative, the term $E_\delta [w(\mu_H, \delta) - w(\mu_L, \delta)]$ is positive, and by the first-order optimality condition in (5.6), we deduce that $\frac{d}{d\alpha} s^*_H > 0$ because

$$
- \frac{c \gamma s^*_H}{(p(1-s^*_H) - c)(1-s^*_H)} + \lambda \left( K + \gamma \ln \left( \frac{p(1-s^*_H)}{p(1-s^*_H) - c} \right) - \gamma E_\delta [w(\mu_H, \delta)] \right) > 0.
$$

Based on (5.3), the first-order derivative of $y^*_H$ with respect to $\alpha$ is

$$
\frac{d}{d\alpha} y^*_H = \frac{d}{ds^*_H} E_\delta \left[ \pi^*_H(K, y^*_H, s^*_H) \right]
$$

$$
\frac{d}{d\alpha} s^*_H > 0.
$$

Hence, both $s^*_H$ and $y^*_H$ are increasing in $\alpha$. With regard to $s^*_L$, note that

$$
\frac{d}{d\alpha} s^*_L = -\frac{\frac{d^2}{ds^2} R(s^*_H, s^*_L)}{\frac{d^2}{ds^2} R(s^*_H, s^*_L)}
$$

$$
= \frac{-1}{\frac{d^2}{ds^2} R(s^*_H, s^*_L)} \left\{ \lambda \left( K + \gamma \ln \left( \frac{p(1-s^*_H)}{p(1-s^*_H) - c} \right) - \gamma E_\delta [w(\mu_L, \delta)] \right) \right\}
$$

$$
- \frac{c \gamma s^*_H}{(p(1-s^*_H) - c)(1-s^*_H)} \right\}
$$

where
Since \( s^*_L \) satisfies the first-order optimality condition in (5.7), we have \( \frac{d}{d\alpha} s^*_L = 0 \). Based on (5.4), the first-order derivative of \( y^*_L \) with respect to \( \alpha \) is

\[
\frac{d}{d\alpha} y^*_L = \left( \frac{d}{ds_H} E_{s_H} \left[ \pi_H^I(K, y^*_H, s^*_H) \right] - \frac{d}{ds_L} E_{s_L} \left[ \pi_L^I(K, y^*_L, s^*_L) \right] \right) \frac{d}{d\alpha} s^*_H + \frac{d}{ds_L} E_{s_L} \left[ \pi_L^I(K, y^*_L, s^*_L) \right] \frac{d}{d\alpha} s^*_L < 0.
\]

Hence, \( y^*_L \) is decreasing in \( \alpha \), but \( s^*_L \) does not change in \( \alpha \).

Change in \( K \) (Proposition 10). The first-order derivative of \( s^*_H \) with respect to \( K \) is

\[
\frac{d}{dK} s^*_H = -\frac{d^2}{ds_H ds_K} R(s^*_H, s_L) \frac{d}{dK} s^*_H = \frac{-1}{d^2 s^*_H R(s^*_H, s_L)} \left\{ p \left( \lambda - \lambda E_{\delta} \left( \frac{\exp(\alpha K)}{\exp(\omega(\mu, \delta))} \right) \right) + \frac{4(1 - \alpha)(1 + \lambda)}{\gamma \prod_{\mu \in \{H, L\}} \prod_{\mu \in \{H, L\}} \exp(\omega(\mu, \delta))} \right\},
\]

where

\[
T = \sinh \left( \frac{\mu_H - \mu_L}{2\gamma} \right) \left( \cosh \left( \frac{\mu_H - \mu_L}{\gamma} \right) + E_{\delta} \left( \cosh \left( \frac{-2K + \mu_H + \mu_L + 2\delta_H + 2\delta_L - 2\delta}{2\gamma} \right) \right) \right) > 0.
\]

In this first-order derivative above, all terms are positive, and thus \( \frac{d}{dK} s^*_H > 0 \). Now, the first-order derivative of \( y^*_H \) with respect to \( K \) is

\[
\frac{d}{dK} y^*_H = \frac{d}{ds_H} E_{s_L} \left[ \pi_L^I(K, y^*_L, s^*_L) \right] \frac{d}{ds_H} s^*_H + \frac{(1 + \lambda) ps s^*_H}{\prod_{\mu \in \{H, L\}} \exp(\omega(\mu, \delta))} \left( \exp \left( \frac{\mu_H + \delta_H + \delta_L}{\gamma} \right) + E_{\delta} \left( \exp \left( \frac{K + \delta}{\gamma} \right) \right) \right) > 0.
\]

Thus, \( y^*_H \) is increasing in \( K \). The first-order derivative of \( s^*_L \) with respect to \( K \) is

\[
\frac{d}{dK} s^*_L = -\frac{d^2}{ds_L ds_K} R(s^*_H, s^*_L) \frac{d}{dK} s^*_L = \frac{-1}{d^2 s^*_L R(s^*_H, s^*_L)} \left\{ p(1 - \alpha) \left( \lambda - \lambda E_{\delta} \left( \frac{\exp(\alpha K)}{\exp(\omega(\mu, \delta))} \right) \right) \right\} > 0.
\]

Finally, the first-order derivative of \( y^*_L \) with respect to \( K \) is

\[
\frac{d}{dK} y^*_L = \frac{d}{ds_L} E_{s_L} \left[ \pi_L^I(K, y^*_L, s^*_L) \right] \frac{d}{ds_L} s^*_L + \frac{(1 + \lambda) ps s^*_H}{\prod_{\mu \in \{H, L\}} \exp(\omega(\mu, \delta))} \left( \exp \left( \frac{\mu_H + \delta_H + \delta_L}{\gamma} \right) + E_{\delta} \left( \exp \left( \frac{K + \delta}{\gamma} \right) \right) \right) + \frac{(1 + \lambda) ps (s^*_L - s^*_H)}{\prod_{\mu \in \{H, L\}} \exp(\omega(\mu, \delta))} \left( \exp \left( \frac{\mu_H + \delta_H + \delta_L}{\gamma} \right) + E_{\delta} \left( \exp \left( \frac{K + \delta}{\gamma} \right) \right) \right) > 0.
\]

Hence, both \( s^*_L \) and \( y^*_L \) are increasing in \( K \).

Change in \( \mu_H \) (Proposition 11). First, the first-order derivative of \( s^*_H \) with respect to \( \mu_H \) is

\[
\frac{d}{d\mu_H} s^*_H = -\frac{d^2}{d\mu_H d\mu_H} R(s^*_H, s_L) \frac{d}{d\mu_H} s^*_H = \frac{1}{d^2 s^*_H R(s^*_H, s_L)} \left\{ (1 - \alpha)(1 + \lambda) E_{\delta} \left( \frac{\exp(\mu_H + \delta)}{\gamma \exp(\omega(\mu, \delta))} \right) + \alpha \lambda ps \exp(\mu_H + \delta) \left( \frac{\exp(\mu_H + \delta)}{\exp(\omega(\mu, \delta))} \right) \right\} < 0.
\]
The first-order derivative of \( y_H^* \) with respect to \( \mu_H \) is
\[
\frac{d}{d\mu_H} y_H^* = \frac{d}{ds_H} E_{\delta,Z}[\pi_H^i(K, y_H^*, s_H^*)] \frac{d}{d\mu_H} s_H^* \\
- \frac{(1 + \lambda) \rho p s_H^* \exp \left( \frac{\mu_H}{\gamma} \right) \left( \exp \left( \frac{\mu_H + \delta H + \delta L}{\gamma} \right) + E_\delta \left[ \exp \left( \frac{K + \delta}{\gamma} \right) \right] \right)}{\prod_{j \in \{H,L\}} \left[ \exp \left( w(\mu_H, \delta) \right) \right]} < 0.
\]

Thus, both \( s_H^* \) and \( y_H^* \) is decreasing in \( \mu_H \).

With regard to \( s_L^* \), its first-order derivative with respect to \( \mu_H \) is
\[
\frac{d}{d\mu_H} s_L^* = -\frac{\partial^2}{\partial s_H \partial \mu_H} R(s_H, s_L^*) = 0.
\]

Thus, \( s_L^* \) is irrelevant to \( \mu_H \). Next, the first-order derivative of \( y_L^* \) with respect to \( \mu_H \) is
\[
\frac{d}{d\mu_H} y_L^* = \rho p (1 + \lambda) \gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)] < 0.
\]

Hence, \( y_L^* \) is decreasing in \( \mu_H \) but \( s_L^* \) does not change in \( \mu_H \).

Change in \( \mu_L \) (Proposition 11). The first-order derivative of \( s_H^* \) with respect to \( \mu_L \) is
\[
\frac{d}{d\mu_L} s_H^* = \frac{-\partial^2}{\partial s_H \partial \mu_L} R(s_H, s_L^*) = \frac{-1}{\partial^2 s_L \partial \mu_H} R(s_H, s_L^*) \left\{ (1 - \alpha)(1 + \lambda) E_\delta \left[ \exp \left( \frac{\mu_L + \delta}{\gamma} \right) \right] \right\} > 0.
\]

The first-order derivative of \( y_H^* \) with respect to \( \mu_L \) is
\[
\frac{d}{d\mu_L} y_H^* = \frac{d}{ds_H} E_{\delta,Z}[\pi_H^i(K, y_H^*, s_H^*)] \frac{d}{d\mu_L} s_H^* > 0.
\]

Thus, both \( s_H^* \) and \( y_H^* \) are increasing in \( \mu_L \). The first-order derivative of \( s_L^* \) with respect to \( \mu_L \) is
\[
\frac{d}{d\mu_L} s_L^* = -\frac{\partial^2}{\partial s_H \partial \mu_L} R(s_H, s_L^*) = -(1 - \alpha) \lambda \rho p E_\delta \left[ \exp \left( \frac{\mu_H + \delta}{\gamma} \right) \right] \left[ \exp \left( \frac{w(\mu_H, \delta)}{\gamma} \right) \right] < 0.
\]

The first-order derivative of \( y_L^* \) with respect to \( \mu_L \) is
\[
\frac{d}{d\mu_L} y_L^* = \frac{d}{ds_L} E_{\delta,Z}[\pi_H^i(K, y_L^*, s_L^*)] \frac{d}{d\mu_L} s_L^* \\
- \frac{(1 + \lambda) \rho p (s_L^* - s_H^*) \exp \left( \frac{\mu_H}{\gamma} \right) \left( \exp \left( \frac{\mu_L + \delta H + \delta L}{\gamma} \right) + E_\delta \left[ \exp \left( \frac{K + \delta}{\gamma} \right) \right] \right)}{\prod_{j \in \{H,L\}} \left[ \exp \left( w(\mu_L, \delta) \right) \right]} < 0.
\]

As a result, both \( s_L^* \) and \( y_L^* \) are decreasing in \( \mu_L \).
Change in $\rho$ (Proposition 12). First, the first-order derivative of $s^*_H$ with respect to $\rho$ is

$$
\frac{d}{d\rho} s^*_H = -\frac{d^2}{dsLd\rho} R(s^*_H, s_L) = \frac{\frac{d^2}{dsLd\rho} R(s^*_H, s_L)}{\frac{d^2}{dsL^2} R(s^*_H, s_L)} = -1\frac{\frac{d^2}{dsLd\rho} R(s^*_H, s_L)}{\frac{d^2}{dsL^2} R(s^*_H, s_L)} p \left\{ (1 - \alpha) (1 + \lambda) \gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)] - \alpha \frac{c s^*_H}{(p(1 - s^*_H) - c)(1 - s^*_H)} \right.
$$

$$
+ \alpha \lambda \left( K + \gamma \ln \left[ \frac{p(1 - s^*_H)}{p(1 - s^*_L) - c} \right] - \gamma E_\delta [w(\mu_H, \delta)] \right) \right\}.
$$

Since $s^*_H$ satisfies the first-order optimality condition in (5.6), we have $\frac{d}{d\rho} s^*_H = 0$. Now, the first-order derivative of $y^*_H$ with respect to $\rho$ is

$$
\frac{d}{d\rho} y^*_H = (1 + \lambda) E_\delta \left[ (p(1 - s^*_H) - c) \left( \mu_H + \delta + \gamma \ln \left( \frac{p(1 - s^*_H)}{c} \right) \right) - p(1 - s^*_H) \gamma \ln \left( \frac{p(1 - s^*_H)}{c} \right) \right]
$$

$$
+ p s^*_H [\mu_H + \delta + K - \gamma w(\mu_H, \delta)] - (p - c) \left( \mu_L + \delta + \gamma \ln \left( \frac{p c}{c} \right) \right) - p \gamma \ln \left( \frac{c}{c} \right) > 0.
$$

Consequently, $y^*_H$ is increasing in $\rho$, but $s^*_H$ does not change in $\rho$. Next, the first-order derivative of $s^*_L$ with respect to $\rho$ is

$$
\frac{d}{d\rho} s^*_L = -\frac{d^2}{dsLd\rho} R(s_H, s^*_L) = \frac{\frac{d^2}{dsLd\rho} R(s_H, s^*_L)}{\frac{d^2}{dsL^2} R(s_H, s^*_L)} = \alpha \rho \left\{ K + \gamma \ln \left[ \frac{p(1 - s^*_L)}{p(1 - s^*_L) - c} \right] - \gamma E_\delta [w(\mu_H, \delta)] \right\}.
$$

Similarly, because $s^*_L$ satisfies the first-order optimality condition in (5.7), we deduce that $\frac{d}{d\rho} s^*_L = 0$.

The first-order derivative of $y^*_L$ with respect to $\rho$ is

$$
\frac{d}{d\rho} y^*_L = \frac{d}{d\rho} g_H(s^*_H) + (1 + \lambda) E_\delta \left[ \left( p(1 - s^*_L) - c \right) \left( \mu_L + \delta + \gamma \ln \left( \frac{p(1 - s^*_L)}{c} \right) \right) - p(1 - s^*_L) \gamma \ln \left( \frac{p(1 - s^*_L)}{c} \right) \right]
$$

$$
- (p(1 - s^*_H) - c) \left( \mu_L + \delta + \gamma \ln \left( \frac{p(1 - s^*_H)}{c} \right) \right) + p(1 - s^*_H) \gamma \ln \left( \frac{p(1 - s^*_H)}{c} \right)
$$

$$
+ p (s^*_L - s^*_H) [\mu_L + \delta + K - \gamma w(\mu_L, \delta)] > 0.
$$

Thus, $y^*_L$ is increasing in $\rho$, but $s^*_L$ does not change in $\rho$.

Change in $\lambda$ (Proposition 13). The first-order derivative of $s^*_H$ with respect to $\lambda$ is

$$
\frac{d}{d\lambda} s^*_H = -\frac{d^2}{ds_L d\lambda} R(s^*_H, s_L) = \frac{\frac{d^2}{ds_L d\lambda} R(s^*_H, s_L)}{\frac{d^2}{ds_L^2} R(s^*_H, s_L)} = -1\frac{\frac{d^2}{ds_L d\lambda} R(s^*_H, s_L)}{\frac{d^2}{ds_L^2} R(s^*_H, s_L)} p \left\{ (1 - \alpha) \gamma E_\delta [w(\mu_L, \delta) - w(\mu_H, \delta)]
$$

$$
+ \alpha \left( K + \gamma \ln \left[ \frac{p(1 - s^*_H)}{p(1 - s^*_L) - c} \right] - \gamma E_\delta [w(\mu_H, \delta)] \right) \right\} > 0.
$$
The first-order derivative of $y_H^*$ with respect to $\lambda$ is:

$$
\frac{d}{d\lambda} y_H^* = \rho E_\delta \left[ (p(1-s_H^*) - c) \left[ \mu_H + \delta + \gamma \ln \left( \frac{p(1-s_H^*) - c}{c} \right) \right] - p(1-s_H^*) \gamma \ln \left( \frac{p(1-s_H^*)}{c} \right) \right. \\
+ ps_H^* \left[ \mu_H + \delta + K - \gamma w(\mu_H, \delta) \right] - \left( (p - c) \left[ \mu_H + \delta + \gamma \ln \left( \frac{p - c}{c} \right) \right] - p \gamma \ln \left( \frac{p - c}{c} \right) \right] \\
\left. + \frac{d}{ds_H} E_{\delta, Z}[\pi_H^*(K, y_H^*, s_H^*)] \frac{d}{d\lambda} s_H^* \right] > 0.
$$

Hence, both $y_H^*$ and $s_H^*$ are increasing in $\lambda$. The first-order derivative of $s_L^*$ with respect to $\lambda$ is

$$
\frac{d}{d\lambda} s_L^* = -\frac{d^2}{ds_L^*} R(s_H, s_L^*) = \alpha p \left\{ \left( K + \gamma \ln \left[ \frac{p(1-s_L^*)}{p(1-s_L^*)-c} \right] - \gamma E_\delta [w(\mu_L, \delta)] \right) \right\} > 0.
$$

Finally, the first-order derivative of $y_L^*$ with respect to $\lambda$ is

$$
\frac{d}{d\lambda} y_L^* = \frac{d}{d\lambda} y_H^* + \rho E_\delta \left[ (p(1-s_L^*) - c) \left[ \mu_L + \delta + \gamma \ln \left( \frac{p(1-s_L^*) - c}{c} \right) \right] - p(1-s_L^*) \gamma \ln \left( \frac{p(1-s_L^*)}{c} \right) \right. \\
- \left( (p - c) \left[ \mu_L + \delta + \gamma \ln \left( \frac{p - c}{c} \right) \right] - p \gamma \ln \left( \frac{p - c}{c} \right) \right] \\
\left. + p(s_L^* - s_H^*) \left[ \mu_L + \delta + K - \gamma w(\mu_L, \delta) \right] + \frac{d}{ds_L} E_{\delta, Z}[\pi_L^*(K, y_L^*, s_L^*)] \right] \frac{d}{d\lambda} s_L^* > 0.
$$

As a result, both $y_L^*$ and $s_L^*$ are increasing in $\lambda$. Q.E.D.

**Proof of Proposition 14.** For this proof, we first define the firm’s profit function in different cases of ignorance. Let $\pi_M^i(K, y, s)$ and $\pi_M^N(K, y, s)$ be the profit function of the firm having the early demand forecast $\bar{\mu} = \alpha \mu_H + (1-\alpha) \mu_L$ with and without insurance, respectively. For $i \in \{H, L\}$, let $E_Z[\pi_M^i(K, y, s)]$ and $E_Z[\pi_M^N(K)]$ denote the firm’s expected profit with and without insurance, respectively, given $\mu = \mu_i$ and $\delta = 0$. Finally, let $E_Z[\pi_M^i(K, y, s)]$ and $E_Z[\pi_M^N(K)]$ denote the firm’s profit functions with and without insurance given $\mu = \bar{\mu}$ and $\delta = 0$. For further details on these and other related expressions, please see Appendix D.

When the insurer ignores adverse selection, it assumes the firm only observes the early demand forecast $\bar{\mu}$ and the insurer offers the insurance contract $(y^{IA}, s^{IA})$ to that firm. In this case, we have

$$
E_{\delta, Z}[\pi_M^i(K, y^{IA}, s^{IA})] - E_{\delta, Z}[\pi_M^N(K)] = 0.
$$

By Proposition 4, we also have

$$
E_{\delta, Z}[\pi_L^i(K, y^{IA}, s^{IA})] - E_{\delta, Z}[\pi_L^N(K)] \geq 0 \geq E_{\delta, Z}[\pi_L^i(K, y^{IA}, s^{IA})] - E_{\delta, Z}[\pi_L^H(K)].
$$

This inequality implies that the contract $(y^{IA}, s^{IA})$ violates the constraint (IR-H) in the initial insurance design problem. Thus, the high-demand firm will not purchase the insurance, whereas the low-demand firm will purchase it. This proves part (i) of the proposition.
For part (ii) of the proposition, we first show that the firm’s net profit from BI insurance, namely \( \pi^I_{ij1}(K, y, s) - \pi^N_{ij1}(K) \), is concave in \( \delta_j \). For all \( i, j \in \{H, L\} \),

\[
\frac{d^2}{d\delta_j^2} \{ \pi^I_{ij1}(K, y, s) - \pi^N_{ij1}(K) \} = - \frac{p s \sech^2 \left( \frac{-K + \mu_i + \delta_j}{2\gamma} \right)}{4\gamma} < 0.
\]

Based on this, we have the following for all \( i \in \{H, L\} \):

\[
E_{\delta, Z}[\pi^I_i(K, y, s)] - E_{\delta, Z}[\pi^N_i(K)] < E_Z[\pi^I_i(K, y, s)] - E_Z[\pi^N_i(K)].
\]

Regarding the contract \((y^{LM}_H, s^{LM}_H)\), note that the constraint (IR-H) is binding in the insurance design problem (IM), which implies that

\[
E_{\delta, Z}[\pi^I_H(K, y^{LM}_H, s^{LM}_H)] - E_{\delta, Z}[\pi^N_H(K)] < E_Z[\pi^I_H(K, y^{LM}_H, s^{LM}_H)] - E_Z[\pi^N_H(K)] = 0.
\]

Therefore, the contract \((y^{LM}_H, s^{LM}_H)\) violates the constraint (IR-H) in the original insurance design problem; hence, the high-demand firm will not purchase the insurance. Similarly, we deduce that the constraint (IR-L) is also violated. Next, we will show that \( \pi^I_{L,j1}(K, y_L, s_L) - \pi^I_{L,j1}(K, y_H, s_H) \) is concave in \( \delta_j \). For all \( j \in \{H, L\} \),

\[
\frac{d^2}{d\delta_j^2} \{ \pi^I_{L,j1}(K, y_L, s_L) - \pi^I_{L,j1}(K, y_H, s_H) \} = - \frac{p(s_H - s_L) \sech^2 \left( \frac{-K + \mu_L + \delta_j}{2\gamma} \right)}{4\gamma} < 0.
\]

Thus, regarding the constraint (IC-LH), we have

\[
E_{\delta, Z}[\pi^I_L(K, y^{LM}_L, s^{LM}_L)] - E_{\delta, Z}[\pi^I_L(K, y^{LM}_H, s^{LM}_H)] < E_Z[\pi^I_L(K, y^{LM}_L, s^{LM}_L)] - E_Z[\pi^I_L(K, y^{LM}_H, s^{LM}_H)] = 0.
\]

In the original problem, the contracts \((y^{LM}_L, s^{LM}_L)\) and \((y^{LM}_H, s^{LM}_H)\) violate the constraint (IC-LH). Therefore, the low-demand firm would have higher profit under the contract \((y^{LM}_L, s^{LM}_L)\). As a result, it would never choose \((y^{LM}_L, s^{LM}_L)\), and it would choose \((y^{LM}_H, s^{LM}_H)\) if this contract is profitable.

Lastly, regarding part (iii) of the proposition, we have the following for the high-demand firm:

\[
0 = E_Z[\pi^I_M(K, y^{IB}, s^{IB})] - E_Z[\pi^N_M(K)] > E_Z[\pi^I_H(K, y^{IB}, s^{IB})] - E_Z[\pi^N_H(K)]
\]

\[
> E_{\delta, Z}[\pi^I_H(K, y^{IB}, s^{IB})] - E_{\delta, Z}[\pi^N_H(K)].
\]

Therefore, the high-demand firm will not purchase the insurance. For the low-demand firm, note that

\[
E_Z[\pi^I_L(K, y^{IB}, s^{IB})] - E_Z[\pi^N_L(K)] > E_Z[\pi^I_M(K, y^{IB}, s^{IB})] - E_Z[\pi^N_M(K)] = 0,
\]

\[
E_Z[\pi^I_L(K, y^{IB}, s^{IB})] - E_Z[\pi^N_L(K)] > E_{\delta, Z}[\pi^I_L(K, y^{IB}, s^{IB})] - E_{\delta, Z}[\pi^N_L(K)].
\]
Thus, \( E_{\delta,Z}[\pi^I_L(K, y^{IB}, s^{IB})] - E_{\delta,Z}[\pi^N_L(K)] \) may be positive, and in that case, the low-demand firm will purchase this insurance contract when the net value is positive.

**Proof of Proposition 15.** Note that the actual profit functions in (6.3)-(6.5) have similar structures, which include the insurer’s profit from the low-demand firm. We let \( R_L(y_L, s_L) \) denote the insurer’s profit function from the low-demand firm, where \( R_L(y_L, s_L) = R(0, 0, y_L, s_L) \). Suppose the insurer only provides a contract to the low-demand firm to maximize \( K > \mu_L \) of the actual profits in (\( L \)). Thus, the profit function can be expressed as a function of \( s_L \).

**First, under this contract, the constraint (IR-L) is binding, and we can express the premium \( y_L \) in a function of \( s_L \). We let \( y_L = \tilde{g}_L(s_L) \), where**

\[
\tilde{g}_L(s_L) = (1 + \lambda) \rho E_{\delta} \left[ \left( p(1 - s_L) - c \right) \left( \mu_L + \delta + \gamma \ln \left( \frac{p(1 - s_L) - c}{\epsilon} \right) \right) - p(1 - s_L) \gamma \ln \left( \frac{p(1 - s_L)}{\epsilon} \right) + p s_L [\mu_L + \delta + K - \gamma w(\mu_L, \delta)] - \left( (p - c) \left[ \mu_L + \delta + \gamma \ln \left( \frac{p - c}{\epsilon} \right) \right] - p \gamma \ln \left( \frac{p}{\epsilon} \right) \right) \right].
\]

**Thus, the profit function can be expressed as a function of \( s_L \). Given the condition \( K > \mu_L + \delta + \gamma \ln \left( \frac{p - c}{\epsilon} \right) \), we know that the profit function is quasi-concave in \( s_L \) and the optimal \( \tilde{s}_L = \min \{ \tilde{s}_L, \bar{s} \} \), where \( \tilde{s}_L \) satisfies the following first-order optimality condition:**

\[
\lambda \left\{ K + \gamma \ln \left[ \frac{p(1 - \tilde{s}_L)}{p(1 - \tilde{s}_L)} \right] - \gamma E_{\delta} \left[ w(\mu_L, \delta) \right] \right\} - \frac{c \gamma \tilde{s}_L}{(p(1 - \tilde{s}_L) - \delta)(1 - \tilde{s}_L)} = 0.
\]

**This first-order optimality condition is the same as the first-order optimality condition of \( s_L^* \) in (5.7). Thus, \( \tilde{s}_L = s_L^* \) and the contracts \( (\tilde{y}_L, \tilde{s}_L) \) and \( (y_L^*, s_L^*) \) differ only different in terms of their premiums \( \tilde{y}_L \) and \( y_L^* \).**

**Now, we compare the profit under the optimal contracts \( (y_L^*, s_L^*) \) and \( (\tilde{y}_L, \tilde{s}_L) \). We let \( L \) denote the difference between two profit functions; i.e.,**

\[
L = R(y_H^*, s_H^*, y_L^*, s_L^*) - R_L(\tilde{y}_L, \tilde{s}_L)
\]
\[ = \alpha \{ y_H^* - \rho (\beta s_H^* p E_s [\min (D_{HH}, K) - \min (D_{HL}, L_H^*(s_H^*) K)] + (1 - \beta) s_H^* p E_s [\min (D_{HL}, K) - \min (D_{HL}, L_H^*(s_H^*) K)]) \) + (1 - \alpha) (1 + \lambda) s_H^* \gamma E_0 [w(\mu_H, \delta) - w(\mu_L, \delta)] \} . \]

Given \((y^*_L, s^*_L)\) and \((\tilde{y}_L, \tilde{s}_L)\), the difference \(R(y_H, s_H, y^*_L, s^*_L) - R_L(\tilde{y}_L, \tilde{s}_L)\) can be expressed as a function of \(s_H\). By the arguments used in the proof of Proposition 6, this difference function is quasi-concave in \(s_H\) and the first-order optimality condition in \(s_H\) is equivalent to (5.6). Therefore, \(L = R(y_H^*, s_H^*, y_L^*, s_L^*) - R_L(\tilde{y}_L, \tilde{s}_L)\) is the maximal difference between the profit functions and the difference is always non-negative. Moreover, because \(R_L(\tilde{y}_L, \tilde{s}_L)\) is the upper bound of the actual profit under each type of ignorance, we have \(\min\{L^{IA}, L^{IM}, L^I\} \geq L\). Q.E.D.

**Proof of Proposition 16.** First, we show that \(s_L^* \geq s_{IA}^* \geq s_H^*\). Considering the first-order optimality conditions in (5.6), (5.7), and (D.8), we have:

\[
\begin{align*}
p p \left\{ (1 - \alpha)(1 + \lambda) &\gamma E_0 [w(\mu_L, \delta) - w(\mu_H, \delta)] - \alpha \frac{c \gamma s_H^*}{(1 - s_H^*)^2 \gamma (1 - s_H^*)} \right. \\
&+ \alpha \lambda \left( K + \gamma \ln \left[ \frac{p(1 - s_L^*)}{p(1 - s_L^*) - c} \right] - \gamma E_0 [w(\mu_H, \delta)] \right) \left. \right\} = 0, \\
\alpha p p \left\{ \lambda \left( K + \gamma \ln \left[ \frac{p(1 - s_L^*)}{p(1 - s_L^*) - c} \right] - \gamma E_0 [w(\mu_L, \delta)] \right) - \frac{c \gamma s_L^*}{(p(1 - s_L^*) - c)(1 - s_L^*)} \right\} = 0, \\
p p \left\{ \lambda \left( K + \gamma \ln \left[ \frac{p(1 - s_{IA}^*)}{p(1 - s_{IA}^*) - c} \right] - \gamma E_0 [w(\mu_L, \delta)] \right) - \frac{c \gamma s_{IA}^*}{(p(1 - s_{IA}^*) - c)(1 - s_{IA}^*)} \right\} = 0.
\end{align*}
\]

Because we have \(E_0 [w(\mu_L, \delta)] \leq E_0 [w(\mu, \delta)] \leq E_0 [w(\mu_H, \delta)]\) and \(E_0 [w(\mu_L, \delta) - w(\mu_H, \delta)] < 0\), the optimal coverage percentages satisfy \(s_L^* \geq s_{IA}^* \geq s_H^*\).

Second, we show that \(s_L^{IM} \geq s_L^*\). By (4.2), we have

\[
\frac{d^2}{d \delta^2} w(\mu, \delta) = \frac{\exp \left( \frac{K + \mu + \delta}{\gamma} \right)}{\left[ \exp \left( \frac{\delta}{\gamma} \right) + \exp \left( \frac{\mu + \delta}{\gamma} \right) \right]^2 \gamma^2} > 0.
\]

Thus, \(w(\mu, \delta)\) is convex in \(\delta\). Now, by the first-order optimality conditions in (5.7) and (D.17), we have

\[
\begin{align*}
\alpha p p \left\{ \lambda \left( K + \gamma \ln \left[ \frac{p(1 - s_L^*)}{p(1 - s_L^*) - c} \right] - \gamma E_0 [w(\mu_L, \delta)] \right) - \frac{c \gamma s_L^*}{(p(1 - s_L^*) - c)(1 - s_L^*)} \right\} = 0, \\
\alpha p p \left\{ \lambda \left( K + \gamma \ln \left[ \frac{p(1 - s_{IA}^*)}{p(1 - s_{IA}^*) - c} \right] - \gamma w(\mu_L, 0) \right) - \frac{c \gamma s_{IA}^*}{(p(1 - s_{IA}^*) - c)(1 - s_{IA}^*)} \right\} = 0.
\end{align*}
\]

Because of the convexity of \(w(\mu, \delta)\) in \(\delta\), we have \(E_0 [w(\mu_L, \delta)] \geq w(\mu_L, 0)\), and thus, \(s_L^{IM} \geq s_L^*\). Based on similar proof arguments, we also have \(s_{IA}^* \geq s_{IA}^*\) and \(s_L^{IM} \geq s_L^{IB} \geq s_H^*\). Q.E.D.
Proof of Proposition 17. First, using the arguments in the proofs in Propositions 4 and 14, we show that given the coverage percentage, the premium is decreasing in $\mu_i$ and concave in $\delta_j$.

To that end, note that

$$\frac{d}{d\mu_i} y = \frac{d}{d\mu_i} \{ E_{\delta,Z}[\pi_i^I(K, y, s)] - E_{\delta,Z}[\pi_i^N(K)] \}$$

$$= \beta \rho \frac{d}{d\mu_i} \{ \pi_{i1}^I(K, y, s) - \pi_{i1}^N(K) \} + (1 - \beta) \rho \frac{d}{d\mu_i} \{ \pi_{ij1}^I(K, y, s) - \pi_{ij1}^N(K) \} > 0,$$

$$\frac{d^2}{d\delta_j^2} y = \frac{d^2}{d\delta_j^2} \{ \pi_{ij1}^I(K, y, s) - \pi_{ij1}^N(K) \} = - \frac{ps \sech^2 \left( \frac{-K + \mu_i + \delta_j}{2\gamma} \right)}{4\gamma} < 0.$$

In addition, by Proposition 2, the premium $y$ should be increasing in $s$, and thus,

$$y^{IB}(s^{IB}) \geq y^{IB}(s^{IA}) \geq y^{IA}(s^{IA}),$$

$$y^{IB}(s^{IB}) \geq y^{IB}(s^{IM}) \geq y^{IM}(s^{IM}).$$

The first inequality in each line is due to the fact that $s^{IB} \geq s^{IA}$ and $s^{IM} \geq s^{IB} \geq s^{IM}$, and the second inequality holds because the premium $y$ is decreasing in $\mu_i$ and concave in $\delta_j$. Q.E.D.

Proof of Proposition 18. By Proposition 1, we know that the optimal recovery effort $e^*_{Lj}(s)$ is non-increasing in $s$. In addition, $s^*_{L} \geq s^{IA}$ and $s^{IB} \geq s^{IA}$ and $s^{IM} \geq s^{IB} \geq s^{IM}$. Based on these facts, we deduce that $e^*_{Lj}(s^{IA}) \leq e^*_{Lj}(s^{IA})$ and $e^*_{Lj}(s^{IB}) \leq \min \{ e^*_{Lj}(s^{IA}), e^*_{Lj}(s^{IM}) \}$. Q.E.D.

When the profit function is not quasi-concave. If the capacity $K$ is sufficiently small, the firm would not benefit from an insurance with low coverage percentage $s$ because it would put the full effort on the recovery and receive no compensation. Thus, with a small $s$, the firm would not purchase insurance and the insurer would make zero profit. As $s$ becomes higher, the firm may purchase the insurance, but this contract may or may not be profitable for the insurer. Figure 6 illustrates that the profit function is not necessarily quasi-concave in these cases.

![Figure 6](image_url)  
**Figure 6** Cross Sections of the Profit Function When the Capacity is Sufficiently Small.

Moreover, if (5.5) is not satisfied, there are four possible shapes for the profit function in terms of $s_H$, which are illustrated in the Figure 7.
Appendix C: Effects of Problem Parameters on the Optimal Contracts

In this section, we provide the numerical results to show how the optimal contracts are affected by the parameters that are not discussed in Section 6.1. First, we are interested in the impacts of the parameters $\beta$, $\delta_H$ and $\delta_L$, which are related to forecast updating. Due to the assumption that $E[\delta] = \beta \delta_H + (1 - \beta) \delta_L = 0$, each of these parameters cannot unilaterally change. Thus, we suppose that there is a fixed range of forecast updates, $G = \delta_H - \delta_L$. If any of $\beta$, $\delta_H$ and $\delta_L$ changes, then the others would also change according to the fixed range. Figures 8-10 show the impacts of these parameters. In these figures, the premiums and the coverage percentage in the optimal contracts appear to be convex in $\beta$, $\delta_H$ and $\delta_L$, but neither increasing nor decreasing.

Figure 8  The Impact of $\beta$ on the Optimal Contracts
Similarly, with regard to the remaining parameters $\gamma$, $p$ and $c$, we illustrate that their impacts on the optimal contracts are not necessarily monotone. For example, Figure 11(a) shows that $y_H^*$ may be increasing or decreasing in $\gamma$; Figure 11(b) shows that $s_H^*$ does not monotonically change in $c$; Figure 11(c) and 11(d) shows that the change in $p$ may have different impacts on the optimal contract $(y_H^*, s_H^*)$, depending on the value of $p$.

**Appendix D: The Contracts under the Ignorance**

In this section, we analyze the insurance contracts when the insurer ignores the information asymmetry, as described in Section 6.2. Based on Table 1, we analyze the contracts under each type of ignorance.
D.1. Ignoring Adverse Selection

We first consider the case that the insurer only ignores the adverse selection. Following the same procedures in the Sections 4 and 5 to model and analyze this problem, we deduce that the firm’s optimal effort without BI insurance is

\[
\hat{e}_{Mj} = \begin{cases} 
\frac{1}{K} F^{-1}_{Mj} \left( \frac{p-e}{p} \right) = \frac{1}{K} \left[ \bar{\mu} + \delta_j + \gamma \ln \left( \frac{p-e}{c} \right) \right] & \text{if } F^{-1}_{Mj} \left( \frac{p-e}{p} \right) < K, \\
1 & \text{otherwise.} 
\end{cases}
\] (D.1)

Therefore, without BI insurance, the expected profits under normal operations and under disruption are, respectively,

\[
\pi^N_{Mj0}(K) = p \{ K + \bar{\mu} + \delta_j - \gamma w(\bar{\mu}, \delta_j) \}, \quad (D.2)
\]

\[
\pi^N_{Mj1}(K) = \pi^N_{Mj1}(K, \hat{e}_{Mj}). \quad (D.3)
\]

Given the expected profits in (D.2) and (D.3), the insurer knows the firm’s expected profit without BI insurance, \( E_{\delta, Z}[\pi^N_{Mj}(K)] \). Similarly, if the firm purchases the BI insurance \((y, s)\), its optimal effort and the expected profits are as follows:

\[
e^*_{Mj}(s) = \begin{cases} 
\frac{1}{K} F^{-1}_{Mj} \left( \frac{p(1-s)-e}{p(1-s)} \right) = \frac{1}{K} \left[ \bar{\mu} + \delta_j + \gamma \ln \left( \frac{p(1-s)-e}{c} \right) \right] & \text{if } F^{-1}_{Mj} \left( \frac{p(1-s)-e}{p(1-s)} \right) < K, \\
1 & \text{otherwise,} 
\end{cases}
\] (D.4)

\[
\pi^I_{Mj0}(K, y, s) = -y + p \{ K + \bar{\mu} + \delta_j - \gamma w(\bar{\mu}, \delta_j) \}, \quad (D.5)
\]

\[
\pi^I_{Mj1}(K, y, s) = \pi^I_{Mj1}(K, e^*_{Mj}(s), y, s). \quad (D.6)
\]
Thus, the insurer also knows the firm’s expected profit with BI insurance, \( E_{\delta,Z}[\pi^I_M(K,y,s)] \). Recall that, in these calculations, the insurer designs an insurance contract for a (fictitious) firm \( M \). In this case, the insurance design problem (IA) is

\[
\max_{(y,s)} \quad y - \rho \{ \beta spE_\epsilon [ \min(D_{MH},K) - \min(D_{MH},e_{MH}^*(s)K) ] \\
+ (1 - \beta)spE_\epsilon [ \min(D_{ML},K) - \min(D_{ML},e_{ML}^*(s)K) ] \}
\]

\[
\text{s.t. } E_{\delta,Z}[\pi^I_M(K,y,s)] \geq E_{\delta,Z}[\pi^N_M(K)] \quad (\text{IR}).
\]

Note that, in the preceding insurance design problem (IA), there is only one IR constraint, and it is always binding under the optimal contract. Therefore, we can express the premium \( y \) as a function of \( s \). Let \( y = g^{IA}(s) \), where

\[
g^{IA}(s) = (1 + \lambda) \rho E_\delta \left[ \left( p(1 - s) - c \right) \left( \bar{\mu} + \delta + \gamma \ln \left( \frac{p(1-s)-c}{c} \right) \right) - p(1-s)\gamma \ln \left( \frac{p(1-s)}{c} \right) \\
+ ps[\bar{\mu} + \delta + K - \gamma w(\bar{\mu},\delta)] - \left( (p - c) \left[ \bar{\mu} + \delta + \gamma \ln \left( \frac{p-s}{c} \right) \right] - p\gamma \ln \left( \frac{p}{c} \right) \right) \right].
\]

Substituting \( g^{IA}(s) \) into the profit function, we can express the objective function only in terms of \( s \). By the arguments used in the proofs of Propositions 6 and 7, we obtain a similar sufficient condition of optimality in \( s \): given that \( K > \bar{\mu} + \delta_H + \gamma \ln \left( \frac{p-s}{c} \right) \), the profit function is quasi-concave in \( s \) and the optimal \( s^{IA} = \min\{s^{IA'}, \bar{s}\} \), where \( s^{IA'} \) satisfies the following first-order optimality condition:

\[
pp \left\{ \lambda \left( K + \gamma \ln \left[ \frac{p(1-s^{IA'})}{p(1-s^{IA'})-c} \right] - \gamma E_\delta [w(\bar{\mu},\delta)] \right) - \frac{c\gamma s^{IA'}}{(p(1-s^{IA'})-c)(1-s^{IA'})} \right\} = 0. \quad (D.8)
\]

The insurer would provide the contract \((y^{IA}, s^{IA})\) which maximizes the profit in the insurance design problem (IA).

**D.2. Ignoring Moral Hazard**

If the insurer ignores moral hazard, the insurer would assume that the firm’s recovery effort is perfectly predictable. In our model, this corresponds to a special case in which \( \delta_H = \delta_L = 0 \). For \( i \in \{H,L\} \), the optimal effort and expected profits of the firm with \( \mu = \mu_i \) and without BI insurance are, respectively,

\[
\hat{e}_i = \begin{cases} 
\frac{1}{\lambda} F_i^{-1} \left( \frac{p-s}{p} \right) = \frac{1}{\lambda} \left[ \mu_i + \gamma \ln \left( \frac{p-s}{c} \right) \right] & \text{if } F_i^{-1} \left( \frac{p-s}{p} \right) < K, \\
1 & \text{otherwise,}
\end{cases}
\]

\[
\pi^N_{i0}(K) = p \{ K + \mu_i - \gamma w(\mu_i,0) \}, \quad (D.10)
\]

\[
\pi^N_{i1}(K) = \pi^N_{i1}(K, \hat{e}_i). \quad (D.11)
\]
Accordingly, the insurer knows the expected profit for each firm without BI insurance, \( E_Z[\pi^N_i(K)] \). If the firm with \( \mu_i \) purchases the BI insurance \((y,s)\), its optimal effort and the expected profits are, respectively,

\[
e_i^*(s) = \begin{cases} 
\frac{1}{K} F_i^{-1} \left( \frac{p(1-s)-c}{p(1-s)} \right) = \frac{1}{K} \left[ \mu_i + \gamma \ln \left( \frac{p(1-s)-c}{c} \right) \right] & \text{if } F_i^{-1} \left( \frac{p(1-s)-c}{p(1-s)} \right) < K, \\
\text{otherwise} & \end{cases}
\] (D.12)

\[
\pi_{i0}^I(K, y, s) = -y + p \{ K + \mu_i - \gamma w(\mu_i, 0) \}, \quad \text{(D.13)}
\]

\[
\pi_{i1}^I(K, y, s) = \pi_{i1}^I(K, e_i^*(s), y, s).
\] (D.14)

Similarly, the insurer also knows the expected profit of each firm with BI insurance, \( E_Z[\pi^I_i(K, y, s)] \). As a result, the insurance design problem \( \text{(IM)} \) is

\[
\max_{(yH,sH),(yL,sL)} \alpha \{ y_H - \rho s_H p E_r \left[ \min \left( D_H, K \right) - \min \left( D_H, e^*_H(s_H)K \right) \right] \} + (1 - \alpha) \{ y_L - \rho s_L p E_r \left[ \min \left( D_L, K \right) - \min \left( D_L, e^*_L(s_L)K \right) \right] \}
\]

s.t.

\[
E_Z[\pi^I_H(K, y_H, s_H)] \geq E_Z[\pi^I_H(K, y_L, s_L)] \quad \text{(IC-HL)},
\]

\[
E_Z[\pi^I_L(K, y_H, s_H)] \geq E_Z[\pi^I_L(K, y_L, s_L)] \quad \text{(IC-LH)},
\]

\[
E_Z[\pi^I_H(K, y_H, s_H)] \geq E_Z[\pi^N_I(K)] \quad \text{(IR-H)},
\]

\[
E_Z[\pi^I_L(K, y_H, s_H)] \geq E_Z[\pi^N_I(K)] \quad \text{(IR-L)}.
\]

By the arguments used the proof of Propositions 6 and 7, we obtain the contracts which optimize the insurer’s profit in this case. Therefore, if \( K > \mu_H + \gamma \ln \left( \frac{p-c}{c} \right) \) and

\[
pp \left\{ (1 - \alpha)(1 + \lambda) \gamma [w(\mu,0) - w(\mu,0)] + \alpha \lambda \left( K + \gamma \ln \left( \frac{p}{p-c} \right) - \gamma w(\mu,0) \right) \right\} \geq 0,
\] (D.15)

then the profit function is quasi-concave in \( s_H \) and \( s_L \). Therefore, the optimal coverages are \( s_H^{LM} = \min \{ s_H^{LM'}, s \} \) and \( s_L^{LM} = \min \{ s_L^{LM'}, s \} \), where \( s_H^{LM'} \) and \( s_L^{LM'} \) satisfy the following first-order optimality conditions:

\[
pp \left\{ (1 - \alpha)(1 + \lambda) \gamma [w(\mu,0) - w(\mu,0)] - \alpha \left( c_{\gamma s_H^{LM'}} \frac{c_{\gamma s_L^{LM'}}}{p(1-s_{LM'})^c} \right) \right\} + \alpha \lambda \left( K + \gamma \ln \left( \frac{p(1-s_{LM'})}{p(1-s_{LM')^c}-c} \right) - \gamma w(\mu,0) \right) = 0,
\] (D.16)

\[
\alpha pp \left\{ \lambda \left( K + \gamma \ln \left( \frac{p(1-s_{LM'})}{p(1-s_{LM')^c}-c} \right) - \gamma w(\mu,0) \right) - \frac{c_{\gamma s_L^{LM'}}}{p(1-s_{LM')^c}-c) (1-s_{LM'})} \right\} = 0.
\] (D.17)
D.3. Ignoring Both Types of Information Asymmetry

In this last case, the firm ignores both adverse selection and moral hazard when designing the insurance contract. Note that this case can be considered as a special case of ignoring adverse selection where we additionally have $\delta_H = \delta_L = 0$. Therefore, the optimal effort and the expected profits for the firm without BI insurance are, respectively,

$$
\hat{e}_M = \begin{cases} 
\frac{1}{K} F^{-1}_M \left( \frac{\mu - c}{p} \right) & \text{if } F^{-1}_M \left( \frac{\mu - c}{p} \right) < K, \\
1 & \text{otherwise},
\end{cases} \quad (D.18)
$$

$$
\pi_{M0}^N(K) = p \{ K + \mu - \gamma w(\mu, 0) \}, \quad (D.19)
$$

$$
\pi_{M1}^N(K) = \pi_{M1}^N(K, \hat{e}_M). \quad (D.20)
$$

Based on (D.19) and (D.20), the insurer know the firm’s expected profit without BI insurance, $E_Z[\pi_M^N(K)]$. On the other hand, if the firm purchases the BI insurance $(y, s)$, its optimal effort and the expected profits are, respectively,

$$
e^*_M(s) = \begin{cases} 
\frac{1}{K} F^{-1}_M \left( \frac{p(1-s)-c}{p(1-s)} \right) & \text{if } F^{-1}_M \left( \frac{p(1-s)-c}{p(1-s)} \right) < K, \\
1 & \text{otherwise},
\end{cases} \quad (D.21)
$$

$$
\pi_{M0}^I(K, y, s) = -y + p \{ K + \mu - \gamma w(\mu, 0) \}, \quad (D.22)
$$

$$
\pi_{M1}^I(K, y, s) = \pi_{M1}^I(K, e^*_M(s), y, s) \quad (D.23)
$$

Therefore, the insurer also knows the firm’s expected profit with BI insurance, $E_Z[\pi_M^N(K, y, s)]$. Consequently, the insurance design problem (IB) is

$$
\max_{(y, s)} \quad y - \rho sp E_x \left[ \min \left( D_M, K \right) - \min \left( D_M, e^*_M(s) K \right) \right] \\
\text{s.t.} \\
E_Z[\pi_M^I(K, y, s)] \geq E_Z[\pi_M^N(K)] \quad (IR).
$$

By the arguments used in the proofs of Propositions 6 and 7, we conclude that, if $K > \bar{\mu} + \gamma \ln \left( \frac{p-c}{c} \right)$, the profit function is quasi-concave in $s$ and the optimal coverage is $s^{IB} = \min \{ s^{IB'}, \bar{s} \}$, where $s^{IB'}$ satisfies

$$
pp \left\{ \lambda \left( K + \gamma \ln \left[ \frac{p(1-s^{IB})}{p(1-s^{IB'})} \right] - \gamma w(\mu, 0) \right) - \frac{c^{IB'}}{(p(1-s^{IB'})-c)(1-s^{IB'})} \right\} = 0. \quad (D.24)
$$