Application to the Motion and Equilibrium of the Planar Kinematic Chains with Rotational Links with Clearances

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Abstract: Based on the algorithm presented in our previous papers [1], we now present their applications to a few problems.

1. Application

We consider the planar kinematical chain from the Figure 1

![Figure 1. Quadrilateral mechanism with clearances in the kinematical joints O₃, O₄.](image)

Determine the equations of motion and the reactions for the mechanism O₁O₂O₃O₄ with clearances in the rotational kinematical joints O₃, O₄, Figure 1, knowing that the element O₁O₂ rotates with constant angular speed ω and the element O₃O₄ is acted by a torque with the moment M. We consider that the elements of the mechanism are homogeneous bar of constant cross section and we consider known the following parameters:

- dimensions l₁ = O₁O₂, l₂ = O₂O₃, l₃ = O₃O₄, l₁ = l₂;
- the coordinates X₀₄, Y₀₄ = 0 of the articulation O₄;
- the clearances r₃, r₄ at the joints O₃, O₄;
- the masses m₃, m₄ and the moments of inertia J₂, J₃;
- the initial conditions, Figure. 2, at t = 0:

\[
X_{O₄} = l₁ + 2l₂ \cos \alpha + r₃ + r₄, \quad \theta₂ = \alpha, \\
X₂ = l₁ + \frac{l₂}{2} \cos \alpha, \quad Y₂ = \frac{l₂}{2} \sin \alpha, \\
\theta₃ = -\alpha, \quad X₃ = X_{O₄} - r₄ - \frac{l₂}{2} \cos \alpha,
\]

![Figure 2. Initial position of the quadrilateral mechanism with clearances.](image)
\[ \dot{X}_2 = 0 \quad , \quad \dot{Y}_2 = l_1 \omega \quad , \quad \dot{\theta}_2 = 0 \quad , \\
\dot{X}_3 = \dot{Y}_3 = 0 \quad , \quad \dot{\theta}_3 = 0. \]

Numerical application for \(l_1 = 0.2 \text{ m} \quad , \quad l_2 = l_3 = 0.6 \text{ m} \quad , \quad r_3 = r_4 = 0.005 \text{ m} \quad , \quad m_2 = m_3 = 2 \text{ kg} \quad , \quad J_2 = J_3 = 0.06 \text{ kgm}^2 \quad , \quad M = 40 \text{ Nm} \quad , \quad \omega = 10 \text{ rad/s}. \]

Solution: Based on the previous relations of calculation, we can write in order the equalities \(\tilde{X}_2^{(2)} = -\frac{l_2}{2}, \quad \tilde{Y}_2^{(2)} = 0,\)

\[
\tilde{U}_{2x} = \tilde{x}_2^{(2)} \cos \theta_2 \quad , \quad \tilde{U}_{2y} = \tilde{x}_2^{(2)} \sin \theta_2 \quad ,
\]

\[
\begin{bmatrix}
B^{(2)}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\tilde{U}_{2y} \\
0 & 1 & \tilde{U}_{2x} \end{bmatrix}, \quad x_3^{(2)} = \frac{l_2}{2},
\]

\[y_3^{(2)} = 0 \quad , \quad U_{3x}^{(2)} = x_3^{(2)} \cos \theta_2 \quad , \quad U_{3y}^{(2)} = x_3^{(2)} \sin \theta_2 \quad ,
\]

\[x_3^{(3)} = -\frac{l_2}{2} \quad , \quad U_{3x}^{(3)} = x_3^{(3)} \cos \theta_3 \quad , \quad U_{3y}^{(3)} = x_3^{(3)} \sin \theta_3 \quad ,
\]

\[D_{3x} = \frac{X_3 + U_{3x}^{(3)} - X_2 - U_{3x}^{(2)}}{r_3},
\]

\[D_{3y} = \frac{Y_3 + U_{3y}^{(3)} - Y_2 - U_{3y}^{(2)}}{r_3},
\]

\[
\begin{bmatrix} D_{3x} \\
D_{3y} \end{bmatrix}, \quad \begin{bmatrix} B^{(2)}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -U_{3y}^{(2)} \\
0 & 1 & U_{3x}^{(2)} \end{bmatrix},
\]

\[
\begin{bmatrix} E^{(2)}_3 \end{bmatrix} = \begin{bmatrix} D_{3x} \end{bmatrix}^T \begin{bmatrix} B^{(2)}_3 \end{bmatrix}, \quad \begin{bmatrix} E^{(3)}_3 \end{bmatrix} = \begin{bmatrix} D_{3x} \end{bmatrix}^T \begin{bmatrix} B^{(3)}_3 \end{bmatrix},
\]

\[
x_4^{(3)} = \frac{l_3}{2}, \quad y_4^{(3)} = 0, \quad U_{4x}^{(3)} = x_4^{(3)} \cos \theta_3 \quad , \quad U_{4y}^{(3)} = x_4^{(3)} \sin \theta_3 \quad ,
\]

\[U_{4x}^{(3)} = x_4^{(3)} \sin \theta_3 \quad , \quad U_{4y}^{(3)} = x_4^{(3)} \sin \theta_3 \quad ,
\]

\[
\begin{bmatrix} B^{(3)}_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -U_{4y}^{(3)} \\
0 & 1 & U_{4x}^{(3)} \end{bmatrix},
\]

\[D_{4x} = \frac{X_{4x} - X_3 - U_{4x}^{(3)}}{r_4},
\]

\[D_{4y} = \frac{Y_{4y} - Y_3 - U_{4y}^{(3)}}{r_4}, \quad \begin{bmatrix} D_4 \end{bmatrix} = \begin{bmatrix} D_{4x} \\
D_{4y} \end{bmatrix},
\]

\[
\begin{bmatrix} E^{(3)}_4 \end{bmatrix} = \begin{bmatrix} D_4 \end{bmatrix}^T \begin{bmatrix} B^{(3)}_4 \end{bmatrix},
\]

\[
\begin{bmatrix} F \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} M \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} M \end{bmatrix}^{-1} \begin{bmatrix} F \end{bmatrix} - \begin{bmatrix} C \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} q \end{bmatrix},
\]

\[
\begin{bmatrix} \dot{q} \end{bmatrix} = \begin{bmatrix} M \end{bmatrix}^{-1} \begin{bmatrix} F \end{bmatrix} - \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} R \end{bmatrix},
\]

Further on, with the aid of the notations \(f_i = \begin{bmatrix} f_1 & f_2 & \ldots & f_6 \end{bmatrix}^T = \begin{bmatrix} M \end{bmatrix}^{-1} \begin{bmatrix} F \end{bmatrix} - \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} R \end{bmatrix},\)

\[
P_i = \begin{cases} q_i, & \text{if } 1 \leq i \leq 6; \\
q_i, & \text{if } 7 \leq i \leq 12; \end{cases}
\]

(1) the systems of first order differential equations \(\frac{dp_i}{dt} = \begin{cases} q_i, & \text{if } 1 \leq i \leq 6; \\
q_i, & \text{if } 7 \leq i \leq 12; \end{cases},\)

which, by numerical integration using the
fourth order Runge–Kutta, leads us to the results captured in Figures. 3–12

From the Figure 3 we observe that the variations of the kinematical parameters $O_2, O_3$ in the case with clearance are small relative to the case without clearance. These variations diminish when the clearances $r_3, r_4$ diminish.

**Figure 3.** Time history of the parameters $q_2$ in the cases without and with clearances.

**Figure 4.** Time history of the parameters $q_3$ in the cases without and with clearances.

**Figure 5.** Time history of the parameter $X_2$.

**Figure 6.** Time history of the parameter $Y_2$.

**Figure 7.** Time history of the parameter $X_3$.

**Figure 8.** Time history of the parameter $Y_3$.

**Figure 9.** Time history of the reaction $H_2$.

**Figure 10.** Time history of the reaction $V_2$.

**Figure 11.** Time history of the reaction $N_3$.

**Figure 12.** Time variation of the reaction $N_4$. 
2. Conclusions

In this paper we presented one application concerning the motion and the equilibrium of the planar chains with rotational linkages with clearances. The applications are complete and numerically solved.

The numerical applications solved here confirm the statements mentioned on the algorithm presented in our previous papers.

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References


