

The Effect of Diversification on Price Informativeness and Asset Values*

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Abstract

We study how an asset’s price informativeness and fundamental value depends on an informed investor’s holdings of other, potentially unrelated, assets. If an asset is sold by a concentrated owner without other holdings, the price decline is low since the sale may be motivated by a liquidity shock. A diversified owner has the choice of which assets to sell upon a shock. Thus, a sale is more revealing of poor asset quality, increasing price informativeness. Even though prices are more revealing, the seller may acquire more information under diversification. When asset values are endogenous, diversification increases incentives to improve value since the asset commands a low price if sold.

KEYWORDS: Corporate governance, banks, blockholders, monitoring, intervention, exit, trading, common ownership.

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This paper analyzes how an asset’s price informativeness and fundamental value depend on an informed investor’s holdings of other assets, even if they are unrelated. We show that, if a seller is diversified, a sale is more likely to be driven by information than a liquidity shock, and thus transmits more information into prices. Moreover, even though greater price impact seems to reduce the returns to private information, we show that diversification may increase a seller’s incentive to become informed about her assets. When asset values are endogenous, greater price informativeness in turn increases the incentives to improve asset values, leading to positive real effects of diversification. Our model can be applied to study how the portfolio composition of a shareholder, bank, venture capitalist, or corporate headquarters affects firm managers’ incentives to induce effort.

We start with a model in which asset values and private information are exogenous. As a benchmark, we analyze the case of a concentrated portfolio. A seller owns n units of a single asset; the results are the same if the benchmark is one unit in each of n perfectly correlated assets. She subsequently learns private information on asset value, which can be high or low. She may also suffer a privately-observed liquidity shock that forces her to raise at least a given dollar amount of funds, although she may choose to sell more, or to sell even absent a shock. Examples include an alternative investment opportunity or withdrawals by her end depositors or investors. Based on her private information and liquidity needs, she retains, partially sells, or fully sells her stake. The asset price is set by a competitive buyer who observes the seller’s trade but not asset value.

If the asset turns out to be good (i.e. high-value) but the seller suffers a liquidity shock, she is forced to partially sell it. Thus, if the asset turns out to be bad (i.e. low-value), the seller sells it by the same amount, to disguise the sale as motivated by a shock. As a result, a bad asset does not command too low a price – adverse selection is mild – and a good asset does not always command a high price as it is sometimes sold and pooled with bad assets. Thus, price informativeness is relatively low.

Under a diversified portfolio, the seller owns one unit in each of n uncorrelated assets. Each asset is traded with a separate buyer, who observes trading in only that asset. The key effect of diversification is that it gives the seller both good and bad assets, and thus the choice of which assets to sell upon a shock. If the shock is small, she can satisfy it by selling only bad assets. Then, being sold is inconsistent with the asset being good and the sale being driven

purely by a shock, and so fully reveals the asset as bad.¹ In contrast, a good asset is retained even upon a shock, and thus receives a high price. Prices can thus be fully revealing of asset quality. If the shock is moderate, it cannot be satisfied by selling only bad assets, and so the seller needs to partially sell good assets as well. However, since the buyer knows that the seller fully sells bad assets upon a shock, a good (and thus partially-sold) asset receives a higher price than under concentration. If the shock is large, it forces the seller to sell good assets to the same extent as bad assets – exactly as under concentration – and so diversification does not give her additional flexibility over what to sell.

Overall, price informativeness is the same under diversification and large shocks as under concentration, higher under moderate shocks, and higher still under small shocks. Intuitively, the smaller the shock, the greater the seller’s flexibility over which assets to sell. Thus, she is forced to sell fewer good assets, and so being sold is a greater signal that the asset is bad. Similarly, if all assets were perfectly correlated, holding multiple assets does not increase the seller’s flexibility – indeed, the benchmark is exactly the same if it involves the seller holding one unit in each of n perfectly correlated assets. The key to flexibility is diversification, and the effect of diversification arises even though the seller is risk-neutral, and even though the buyer does not observe the seller’s trades in other assets. Merely knowing that she has other assets in her portfolio, that she could have sold upon a shock, is sufficient for the buyer to give a low price to a sold asset.

Note that adding additional risky assets to the seller’s portfolio is critically different from adding financial slack, i.e. liquid assets (such as Treasury bills) on which the seller has no private information (as studied by Malherbe (2014)) or borrowing capacity. Consider a seller who owns only asset i and adds Treasury bills to her portfolio. Treasury bills provide *uncontingent* liquidity: since there is no private information, she always sells them first. Now assume the seller instead adds asset j to her portfolio. If asset i (j) turns out to be good (bad), then asset j is indeed no different from a Treasury bill – it is sold first and provides liquidity. However, if asset i (j) turns out to be bad (good), then the seller will not sell asset j – it provides no

¹For example, Warren Buffett’s disposal of Exxon Mobil and ConocoPhillips in late 2014 but not Suncor Energy was viewed by the market as a negative signal on the sold companies in particular, rather than purely due to a liquidity shock (e.g. investment opportunities suddenly appearing in non-energy sectors). More broadly, Huang, Ringgenberg, and Zhang (2016) show that upon a liquidity shock, mutual funds do not scale back all positions equally; instead, they disproportionately sell stocks that end up subsequently underperforming – consistent with them having negative information about those stocks.

liquidity, and does not insure her against the need to fully sell asset i , leading to high price informativeness. As a result, price informativeness is higher when adding asset j rather than Treasury bills, even though it provides only *contingent* liquidity.

This baseline model has a number of implications. The price decline upon a sale is stronger when an informed seller owns multiple assets. The “price” can refer either to a trading price, or market perceptions of quality. If a bank sells a loan, that borrower’s perceived creditworthiness falls more if the bank had other loans it could have sold instead. A conglomerate’s decision to exit a business line is a more negative signal of industry prospects than if a focused firm scaled back its operations.

We then extend the model to the case of endogenous information acquisition. Now, the seller is no longer endowed with information about asset values, but pays a cost to acquire it. One might think that the seller acquires less information under diversification, because information is less useful to her for two reasons: first, her information is revealed in prices to a greater extent; second, she can use information to sell only 1 rather than n units in each asset – she is spread more thinly. We show that information acquisition may actually be higher under diversification. The intuition is as follows. Under concentration, if the seller suffers a liquidity shock, she is forced to partially sell her only asset. Knowing whether the asset is good or bad has no value, since she is forced to sell it either way. Under diversification, information tells her which assets are good and bad, and so she is able to satisfy the shock by selling only bad assets. This advantage is particularly important if the liquidity shock is likely, and also if the shock is small so that the seller has a choice of which assets to sell. Even if the liquidity shock is large and infrequent, so that information acquisition is lower under diversification, it may be outweighed by the seller’s trading flexibility so that price informativeness still increases overall.

We finally endogenize asset values to demonstrate the real effects of price informativeness and thus portfolio diversification. Now, the value of each asset depends on an unobservable effort decision by a manager – for example, the asset may be equity or debt in a firm. If the firm’s manager works, the asset is good, else it is bad. The manager is concerned with both fundamental value and the short-term asset price. The threat of selling, and thus receiving a low asset price ex post, induces effort ex ante, as in the “governance through exit” models of Admati and Pfleiderer (2009) and Edmans (2009). Under a concentrated portfolio, effort incentives are low. If the manager works, the seller may suffer a shock and be forced to sell. Thus, the manager suffers a low stock price, which reduces the reward for working. If the

manager shirks, his firm is sold, but he does not suffer too low a price, because the sale is also consistent with a shock. Under diversification, the reward for working is higher. With a small shock, a manager's firm is never sold if he works. With a moderate shock, it is sold but only partially, and so the firm is given a higher price than under concentration. In addition, the punishment for shirking is now higher due to greater price informativeness. With a large shock, the seller is forced to sell all firms fully upon a shock, just as in the a concentrated portfolio case, and so governance is the same.

We discuss several other potential applications. The first is governance through voice, where the action is now taken by the seller herself. This applies to the case in which the seller is an investor who engages in monitoring. As shown by Kahn and Winton (1998) and Maug (1998), monitoring incentives are low under a concentrated portfolio for two reasons. If the investor monitors, she may suffer a shock that forces her to sell prematurely, reducing her payoff to monitoring; if she does not monitor, she may sell ("cut-and-run"), which yields a relatively high price since the sale is also consistent with a shock. Under diversification, an investor does not have to fully sell a monitored firm even if she suffers a shock, increasing the payoff to monitoring, and suffers a low price if she cuts-and-runs. This advantage must be weighed against the fact that diversification reduces the investor's stake in an individual firm, and thus the incentives or ability to monitor. Second, we study the case in which the seller receives a fixed reservation payoff upon sale, independent of the effect of sale on the asset's reputation. The seller is no longer concerned with price impact, and thus camouflaging a sale as motivated by a shock – yet diversification can still increase price informativeness. This model applies to the case of discontinuing a relationship, such as a bank ceasing to lend, a director quitting a board, or a venture capitalist not investing in a future financing round. Third, and relatedly, if the headquarters of a conglomerate or the general partner of a private equity firm invests in many unrelated businesses, this increases the price decline upon sale and thus both the headquarters' and divisional managers' incentives to exert effort.

In addition to these additional applications, we also demonstrate robustness to alternative modeling assumptions. In one, information asymmetry (the difference in valuation between good and bad assets), and thus the price impact of selling, differs across assets. In a second, diversification involves the seller owning two assets, rather than a continuum. In a third, a single buyer observes the seller's trade in all assets. In this case, price informativeness under diversification can be even higher than in the core model because the buyer can engage in

relative performance evaluation – compare the trade in one asset to that in another, to better infer whether any sale of the first asset was due to low quality or a liquidity shock.

Our paper is primarily related to the literature on the price impact of sales, starting with the adverse selection model of Akerlof (1970) and surveyed by Tirole (2011). This literature has shown how price informativeness depends on a number of characteristics of the asset in question.² Our results suggest that price informativeness also depends on other, unrelated assets owned by the same seller. The comparison with the classic microstructure models of Glosten and Milgrom (1985) and Kyle (1985) leads to an interesting intuition. Unlike in those papers, there are no separate noise traders in our setting, but the liquidity shock can be thought of as effectively creating a noise trader – the seller in the liquidity-shock state, with whom the investor is camouflaging. Diversification reduces this camouflage, since a shocked seller need not sell good assets; she can still trade on her information. As a result, an unshocked investor cannot pretend that her trades are not driven by information. Here, a shocked seller’s other assets mean that she is still an informed trader, not a noise trader, and so effectively remove the noise trader from the model.

Closest to our paper are models where price informativeness depends on the probability that the seller trades for non-informational reasons such as an alternative investment opportunity (Myers and Majluf (1984)), diversification (Eisfeldt (2004)) or liquidity shocks (Diamond and Verrecchia (1991)). However, these papers only consider a single asset. Admati (1980), Caballé and Krishnan (1994), Pasquariello (2006), and He (2009) study multi-asset trading models. In those papers, the buyer can learn about asset i ’s payoff by observing the seller’s trade in asset j which is correlated.³ Here, asset j is relevant even though it is uncorrelated, and even if the buyer cannot observe the trade in asset j . DeMarzo (2005) studies an informed seller’s incentives to pool assets before selling securities backed by them. Doing so is analogous to a concentrated portfolio in our model, since pooling does not allow the seller to divest one asset disproportionately. All of these models feature no liquidity shock and thus do not study how

²Examples include the presence of noise traders (Glosten and Milgrom (1985), Kyle (1985)), randomness in asset supply (Grossman and Stiglitz (1980)), the amount of information the seller (Hirshleifer (1971)) and potential purchasers (Plantin (2009)) have about the asset, limited capital among potential informed purchasers (Gromb and Vayanos (2002)), the information sensitivity of the asset ((Myers and Majluf (1984), Gorton and Pennacchi (1990)), the amount that the seller retains of the asset (Leland and Pyle (1977)), future adverse selection in the same asset (Bolton, Santos, and Scheinkman (2009)), and the chain of intermediation between buyers and sellers (Glode and Opp (2016)).

³Technically, in Admati (1985) there is no dedicated buyer or market maker, however, all agents can condition their order-flow on the prices of all traded assets.

multi-asset ownership affects how the investor trades when faced with the shock, and thus the extent to which she can camouflage her trades upon no shock.

The application to endogenous asset values is also related to the governance literature. This literature has highlighted two mechanisms through which investors may exert governance: voice and exit. These literatures have been developed relatively independently⁴, and the determinants of each mechanisms are quite different. We identify a common channel through which diversification can strengthen both mechanisms – increased price informativeness, which mitigates conventional wisdom that diversification necessarily weakens governance by spreading an investor too thinly. In doing so, we also extend governance models to multiple firms. In reality, investors hold sizable stakes in several firms – shareholders own multiple blocks⁵ and banks lend large amounts to multiple borrowers – but most governance models consider a single firm. One exception is the voice model of Admati, Pfleiderer, and Zechner (1994), which features no information asymmetry and instead focuses on the trade-off between risk-sharing and the free-rider problem. Another is Diamond (1984), who shows that diversification reduces the deadweight loss that is required to incentivize the bank to repay its end investors.

1 The Model

This section considers a pure trading model in which both asset values and private information are exogenous, to highlight the effect of portfolio diversification on price informativeness. In Section 2 we endogenize information acquisition and in Section 3 we endogenize asset value.

1.1 Setup

We consider two versions of the model. The first is a preliminary benchmark of a concentrated portfolio. A single seller (“she”) owns a continuum of units of a single asset, of mass n . The second version is the main model of a diversified portfolio, where the seller owns one unit in each of a continuum of assets, each indexed i , of mass n . (Appendix C.1 considers the case of two assets). Note that, in both models, the seller owns the same number (n) of units and

⁴Edmans and Manso (2011), Levit (2013) and Fos and Kahn (2015) feature both mechanisms, but model each using quite different frameworks.

⁵See Antón and Polk (2014), Azar, Schmalz, and Tecu (2017), Hau and Lai (2013), and Kang, Luo, and Na (2017).

thus the same ex ante portfolio value. Let z denote the seller's number of units in a given asset, i.e. $z = n$ (1) under concentrated (diversified) ownership. The results are identical if the benchmark is instead 1 unit in a continuum of n firms with perfectly correlated values. In this case, the comparison of the concentrated and diversified ownership cases holds constant the number of assets owned and only changes their correlation.

The model consists of three periods. At $t = 1$, Nature chooses the fundamental value of each asset i , $v_i \in \{\underline{v}, \bar{v}\}$, where $\bar{v} > \underline{v} > 0$ and $\Delta \equiv \bar{v} - \underline{v} > 0$. v_i are independently and identically distributed (“i.i.d.”) across assets, and $\tau \equiv \Pr[v_i = \bar{v}] \in (0, 1)$ is common knowledge. Due to the law of large numbers, the actual proportion of assets for which $v_i = \bar{v}$ is τ . The seller privately observes v_i under concentration and $\mathbf{v} \equiv [v_i]_{i=0}^n$ under diversification. We use “good” (“bad”) asset to refer to an asset with $v_i = \bar{v}$ (\underline{v}).

At $t = 2$, the seller is subject to a portfolio-wide liquidity shock $\theta \in \{0, L\}$, where $L > 0$ and $\Pr[\theta = L] = \beta \in (0, 1]$. The variable θ is privately observed by the seller and represents the dollar amount of funds that she must raise. If she cannot raise θ , she raises as much as possible. Formally, failing to raise θ imposes a cost $K > 0$ multiplied by the shortfall, which is sufficiently large to induce her to meet the liquidity need to the extent possible. The seller may raise more than θ dollars, i.e. we allow for voluntary sales. Note that the model allows for $\beta = 1$, i.e. common knowledge that the seller has suffered a shock, such as a financial crisis.

After observing the shock, the seller sells $x_i \in [0, z]$ units in asset i . We use “fully sold” to refer to asset i if $x_i = z$ and “partially sold” if $x_i \in (0, z)$. We assume that short selling is either costly or constrained, otherwise the seller's initial position would not matter; for simplicity, we model these costs or constraints by not allowing for short sales. When the asset is a security, some investors (e.g. mutual funds) are constrained from short selling. In addition, our model applies to assets other than securities, e.g. a bank selling loans or a conglomerate selling divisions, where short sales are not possible. Under diversification, if $x_i^* = x_j^* \forall i \neq j$, we say that the seller engages in “balanced exit.” Otherwise, she engages in “imbalanced exit.” The sold units x_i are purchased by the buyer for asset i (“he”). The buyer is competitive and risk-neutral, and thus can be interpreted as a pool of competitive buyers. There is a separate buyer for each asset who observes only x_i and not x_j for $j \neq i$, θ , nor v_i ; Appendix C.2 considers the case of a single buyer for all assets, who observes x_j , $j \neq i$. Each buyer sets the price $p_i(x_i)$ at $t = 2$ to equal the asset's expected value, conditional on the observed trade x_i . We denote $\mathbf{p} \equiv [p_i(x_i)]_{i=0}^n$ and $\mathbf{x} \equiv [x_i]_{i=0}^n$.

At $t = 3$, asset values are realized. The seller's utilities under concentration and diversification are respectively given by:

$$u_I(x_i, v_i, p_i(x_i), \theta) = x_i p_i(x_i) + (n - x_i) v_i - K \times \max\{0, \theta - x_i p_i(x_i)\}. \quad (1)$$

$$u_I(\mathbf{x}, \mathbf{v}, \mathbf{p}, \theta) = \int_0^n [x_i p_i(x_i) + (1 - x_i) v_i] di - K \times \max\left\{0, \theta - \int_0^n x_i p_i(x_i) di\right\} \quad (2)$$

The equilibrium concept we use is Perfect Bayesian Equilibrium. Here, it involves: (i) A trading strategy by the seller that maximizes her expected utility u_I given each buyer's price-setting rule and her private information on \mathbf{v} (v_i) and (ii) a price-setting rule by each buyer that allows him to break even in expectation, given the seller's strategy. Moreover, (iii) each buyer uses Bayes' rule to update his beliefs from the seller's trades, (iv) all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium, and (v) the pricing function is monotonic, i.e. $p_i(x_i)$ is weakly decreasing, holding constant x_j , $j \neq i$.⁶ Since assets are ex-ante identical, we focus on symmetric pure strategy equilibria⁷, in which each buyer uses a symmetric pricing function. We also assume that the seller does not sell a good asset if unshocked. This is intuitive since the price can never exceed the value of a good asset \bar{v} , but simplifies the analysis as we need not consider equilibria under which a good asset is partially sold, but still fully revealed as good as bad assets are sold in greater volume. Prices informativeness are exactly the same without this restriction.

1.2 Trade Under Concentration

Lemma 1 characterizes all equilibria under a concentrated portfolio.⁸

⁶Focusing on weakly decreasing price functions imposes some restrictions on off-equilibrium prices, and thus the amounts sold in equilibrium. However, since these restrictions do not affect on-equilibrium prices, they generally do not affect real actions when introduced in Section 3. In addition, weakly decreasing pricing functions are consistent with other microstructure theories (e.g. Kyle (1985)) and empirical evidence (e.g. Gorton and Pennacchi (1995), Ivashina (2009)).

⁷The results continue to hold if we allow for mixed strategies, although the proofs are much lengthier. Where relevant, we discuss the minor changes that occur when allowing for mixed strategies.

⁸While the prices on the equilibrium path are unique, the prices off-equilibrium are not. The pricing function in equation (4) ensures monotonicity. A similar comment applies to subsequent pricing functions. In addition, equilibria can differ in their on-the-path trading volumes when $L/n > \underline{v}$. Specifically, if $L/n > \underline{v}$ then any $\bar{x}_{con}(\tau) \in [\min\{\frac{nv}{\bar{p}_{con}(\tau)}, n\}, \min\{\frac{L}{\bar{p}_{con}(\tau)}, 1\}]$ can be an equilibrium. In those cases, we select $\bar{x}_{con}(\tau) = \min\{\frac{L}{\bar{p}_{con}(\tau)}, 1\}$. Intuitively, this selection implies that if there is an equilibrium in which the seller can meet her liquidity needs, then selected equilibria must have this property, and if such an equilibrium does not exist, then the selected equilibrium is the one that maximizes the seller's revenue. This selection can be justified

Lemma 1 (*Concentration*): *An equilibrium under a concentrated portfolio always exists. In any equilibrium, the seller's trading strategy is:*

$$x_{con}^*(v_i, \theta) = \begin{cases} 0 & \text{if } v_i = \bar{v} \text{ and } \theta = 0 \\ \bar{x}_{con}(\tau) = n \min\left\{\frac{L/n}{\bar{p}_{con}(\tau)}, 1\right\} & \text{otherwise} \end{cases}, \quad (3)$$

and prices of asset i are:

$$p_i^*(x) = \begin{cases} \bar{v} & \text{if } x = 0 \\ \bar{p}_{con}(\tau) = \underline{v} + \Delta \frac{\beta\tau}{\beta\tau+1-\tau} & \text{if } x \in (0, \bar{x}_{con}(\tau)], \\ \underline{v} & \text{if } x > \bar{x}_{con}(\tau). \end{cases} \quad (4)$$

We will refer to the seller's type as (v_i, θ) , i.e. a pair that indicates her information on the value of asset i and whether she has suffered a liquidity shock. (Sometimes we will define the type as referring only to v_i , in which case it refers to both $(v_i, 0)$ and (v_i, L)).

Equation (3) shows that, if the asset is good and the seller suffered a shock, she sells $\bar{x}_{con}(\tau)$. This quantity is the minimum required to satisfy the shock: if it were greater, type- (\bar{v}, L) would deviate and sell less, retaining more of a good asset and receiving no lower a price (since prices are non-increasing). Thus, if the asset is bad, the seller sells the same amount ($\bar{x}_{con}(\tau)$), to disguise the motive for her sale. The price of a sold asset, $\bar{p}_{con}(\tau)$, is relatively high as the buyer attaches a probability $\frac{\beta\tau}{\beta\tau+1-\tau}$ that the sale was of a good asset and due to a shock. Thus, adverse selection is not so severe under concentration.

Since the buyer breaks even in expectation, the seller's trading gains when selling $\bar{x}_{con}(\tau)$ of a bad asset equal her trading losses when forced to sell $\bar{x}_{con}(\tau)$ of a good asset due to a shock. Thus, the possibility of trade has no effect on her ex ante portfolio value, which is $n(\underline{v} + \Delta\tau)$. In Section 3, when we endogenize v_i , we will show that the possibility of trade changes portfolio value.

using the Grossman and Perry (1986) criterion; moreover, it has no effect on price informativeness or any of our other main results, which apply when $L/n \leq \underline{v}$.

1.3 Trade Under Diversification

Under diversification, the seller decides not only how much of her portfolio to sell, but also which assets. Lemma 2 characterizes all equilibria.

Lemma 2 (*Diversification*): *An equilibrium under a diversified portfolio always exists.*

(i) *If $L/n \leq \underline{v}(1 - \tau)$ then in any equilibrium*

$$x_{div}^*(v_i, \theta) = \begin{cases} 0 & \text{if } v_i = \bar{v} \\ \underline{x}(\theta) \in [\frac{\theta/n}{\underline{v}(1-\tau)}, 1] & \text{if } v_i = \underline{v}, \end{cases} \quad (5)$$

and prices of asset i are:

$$p_i^*(x_i) = \begin{cases} \underline{v} + \Delta \frac{\tau}{\tau + (1-\tau)(1-\beta) \cdot \mathbf{1}_{\underline{x}(0)=0}} & \text{if } x_i = 0 \\ \underline{v} & \text{if } x_i > 0. \end{cases} \quad (6)$$

(ii) *If $\underline{v}(1 - \tau) < L/n < \underline{v}$ then there exists an equilibrium in which*

$$x_{div}^*(v_i, \theta) = \begin{cases} 0 & \text{if } v_i = \bar{v} \text{ and } \theta = 0 \\ \bar{x}_{div}(\tau) = \frac{\underline{v} + \frac{L/n - \underline{v}}{\tau}}{\bar{p}_{div}(\tau)} < 1 & \text{if } v_i = \underline{v} \text{ and } \theta = 0, \text{ or } v_i = \bar{v} \text{ and } \theta = L \\ 1 & \text{if } v_i = \underline{v} \text{ and } \theta = L, \end{cases} \quad (7)$$

and prices of asset i are:

$$p_i^*(x_i) = \begin{cases} \bar{v} & \text{if } x_i = 0 \\ \bar{p}_{div}(\tau) = \underline{v} + \Delta \frac{\beta\tau}{\beta\tau + (1-\beta)(1-\tau)} & \text{if } x_i \in (0, \bar{x}_{div}(\tau)], \\ \underline{v} & \text{if } x_i > \bar{x}_{div}(\tau). \end{cases} \quad (8)$$

(iii) *If $\underline{v} \frac{1-\tau}{\beta\tau + 1 - \tau} \leq L/n$ then there exists an equilibrium as described by Lemma 1 except \bar{x}_{con} is replaced by \bar{x}_{con}/n .*

(iv) *No other equilibrium exists.*

The intuition is as follows. If $L/n \leq \underline{v}(1 - \tau)$, the liquidity shock is sufficiently small that it can be satisfied by selling only bad assets. She thus retains all good assets, regardless of whether she suffers a shock. If there is no shock (w.p. $(1 - \beta)$), the seller no longer has strict incentives to sell bad assets because doing so is fully revealing. Thus, while there is an equilibrium in which the seller at least partially sells all bad assets ($\underline{x}(0) > 0$), there is also an equilibrium in which she full retains them ($\underline{x}(0) = 0$). Overall, bad assets are retained with probability (“w.p.”) $(1 - \beta) \cdot \mathbf{1}_{\underline{x}(0)=0}$; as a result, a retained asset is not fully revealed and only priced at $\underline{v} + \Delta \frac{\tau}{\tau + (1 - \tau)(1 - \beta) \cdot \mathbf{1}_{\underline{x}(0)=0}}$ rather than \bar{v} . Any asset that is at least partially sold is fully revealed as being bad and priced at \underline{v} .⁹

For $\underline{v}(1 - \tau) < L/n < \underline{v}$, the shock is sufficiently large that the seller cannot satisfy it only by fully selling bad assets, but sufficiently small that she can still sell good assets less (engage in imbalanced exit). She sells $\bar{x}_{div}(\tau)$ from each good asset. Thus, upon no shock, she no longer retains bad assets but sells $\bar{x}_{div}(\tau)$ to disguise her sale as that of a good asset driven by a shock. As a result, (\bar{v}, L) is pooled with $(\underline{v}, 0)$.¹⁰ In addition, retained assets are fully revealed as good and priced at \bar{v} .

Finally, for $\underline{v} \frac{1 - \tau}{\beta\tau + 1 - \tau} \leq L/n$, the shock is sufficiently large that it forces the seller to sell good assets as much as bad assets (engage in balanced exit). Thus, (\bar{v}, L) is pooled with not only $(\underline{v}, 0)$ (as in the moderate-shock case) but also (\underline{v}, L) , further reducing its price below \bar{v} and increasing the price of (\underline{v}, L) above \underline{v} . Since the seller’s trading strategy is the same as under concentration ((\bar{v}, L) , $(\underline{v}, 0)$, and (\underline{v}, L) are all pooled), prices are exactly the same.¹¹

We denote the expected equilibrium price of asset i , given value v_i , under diversification and concentration by $P_{div}(v_i, \tau)$ and $P_{con}(v_i, \tau)$, respectively. Proposition 1 gives conditions under which the expected price of a good (bad) asset is higher (lower) under diversification than concentration, i.e. closer to fundamental value so that price informativeness is higher.

⁹If we allow for mixed strategies, in the case of no shock, a seller could fully retain all bad assets with an interior probability, rather than always (if $\underline{x}(0) = 0$) or never ($\underline{x}(0) > 0$). In the case of a shock, a seller could satisfy it by fully selling some bad firms and fully retaining others, rather than partially selling all bad firms. The results are unchanged: the term $\mathbf{1}_{\underline{x}(0)=0}$ is simply replaced by the probability that a bad firm is retained, combining across the cases of a shock and no shock.

¹⁰We continue to use “type” to refer to (v_i, θ) ; this is a slight abuse of terminology since, under diversification, the seller’s type consists of the entire vector of firm values.

¹¹Note that, for $\underline{v} \frac{1 - \tau}{\beta\tau + 1 - \tau} \leq L/n < \underline{v}$, both the imbalanced exit equilibrium of part (ii) and the balanced exit equilibrium of part (iii) can be sustained. While the seller has the option to satisfy a shock by selling bad assets more, she may also sell good assets to the same degree as bad assets. Doing so increases her trading losses on good assets but reduces them on bad assets, since bad assets are now pooled with good assets upon a shock.

Proposition 1 (*Price informativeness*): Suppose $\tau \in (0, 1)$, then:

(i) If $\underline{v}(1 - \tau) < L/n$ or $\frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}} \leq \beta$, then in any equilibrium

$$P_{div}(\bar{v}, \tau) \geq P_{con}(\bar{v}, \tau) \text{ and } P_{div}(\underline{v}, \tau) \leq P_{con}(\underline{v}, \tau), \quad (9)$$

with strict inequalities if $L/n \leq \underline{v} \frac{1-\tau}{\beta\tau+1-\tau}$.

(ii) If $L/n \leq \underline{v}(1 - \tau)$ and $\beta < \frac{\sqrt{1-\tau}}{\sqrt{\tau}+\sqrt{1-\tau}}$, then there is an equilibrium with $\underline{x}(0) = 0$ and

$$P_{div}(\bar{v}, \tau) < P_{con}(\bar{v}, \tau) \text{ and } P_{div}(\underline{v}, \tau) > P_{con}(\underline{v}, \tau). \quad (10)$$

In all other equilibria (i.e. if $\underline{x}(0) > 0$), we have (9).

Under diversification, the seller has a diversified portfolio of good and bad assets. This allows her to choose which assets to sell upon a shock – in particular, she sells bad assets first. In the moderate-shock equilibrium of part (ii) of Lemma 2, a shock causes her to fully sell bad assets and partially retain good assets. Thus, bad assets are fully revealed upon a shock and priced at \underline{v} , when they are always pooled (with (\bar{v}, L) and $(\underline{v}, 0)$) under concentration. As a result, bad assets receive a lower expected price under diversification. One application of the model is to debt or equity securities. Scholes (1972), Mikkelson and Partch (1985), Holthausen, Leftwich, and Mayers (1990), and Sias, Starks, and Titman (2006) show that sales by large shareholders reduce the stock price due to conveying negative information; Dahiya, Puri, and Saunders (2003) find similar results for loan sales. Our model predicts that the price declines upon a sale are greater under diversification.¹² On the other hand, a good asset is retained and thus fully revealed upon no shock. Upon a shock, a good asset is sold, but only partially. The buyer knows that, if the asset were bad and the seller had suffered a shock, it would have been sold fully. Thus, it is priced at $\underline{v} + \Delta \frac{\beta\tau}{\beta\tau+(1-\beta)(1-\tau)}$ (i.e. pooled with only $(\underline{v}, 0)$) rather than $\underline{v} + \Delta \frac{\beta\tau}{\beta\tau+1-\tau}$ (i.e. pooled with $(\underline{v}, 0)$ and (\underline{v}, L)) under concentration.

A similar intuition applies to the small-shock equilibrium of part (i), whether the seller fully retains good assets. As a result, the sale of asset i cannot be attributed to a shock because, if

¹²In He (2009), the price impact of a sale is stronger if the asset is *more* correlated with other assets in the seller's portfolio. Retaining an asset is even more costly when it is positively correlated with the rest of the portfolio, and particularly so when the asset is low-quality. Thus, retention is a stronger signal of asset quality, leading to a steeper pricing function. His model features risk aversion rather than liquidity shocks.

asset i were good and the seller had needed liquidity, she would have sold other assets instead. Thus, a sold asset is fully revealed as being bad and priced at \underline{v} . On the other hand, since sold assets are fully revealed as bad, the seller no longer has strict incentives to sell bad assets when she does not suffer a shock. If she fully retains bad assets ($\underline{x}(0) = 0$) upon no shock (which occurs w.p. $1 - \beta$), then being retained is no longer fully revealing. If β is sufficiently high, this case is infrequent and so price informativeness is higher under diversification in any equilibrium ($P_{div}(\underline{v}, \tau) > P_{con}(\underline{v}, \tau)$ in expression (10)). Even if β is low, price informativeness is higher under the most informative equilibrium under diversification than under concentration, and this is the equilibrium that will be selected under the efficiency criterion (Section 3 shows that, when firm value is endogenous, the most efficient equilibrium involves the highest possible price informativeness.) In the large-shock equilibrium of part (iii), the seller's trading behavior is exactly the same as under concentration, and so price informativeness is no higher.

Note that the effect on price informativeness stems from diversification, rather than simply giving the seller additional assets. If the seller held 1 unit in a continuum of perfectly correlated assets of mass n , prices would be the same as under the benchmark as the seller could not sell bad assets more and good assets less upon a shock – either all assets are good, or all are bad. This result implies that price informativeness is increasing in the diversification of a seller's portfolio. Similarly, diversification alone is insufficient; the seller must have flexibility over which assets to sell. An index fund is diversified, but constrained to selling all assets equally.

In addition, the results show that diversifying by adding additional assets to the seller's portfolio is different from adding financial slack, i.e. liquid assets (such as Treasury bills) on which the seller has no private information, or risk-free borrowing capacity. Consider the effect of adding A dollars of liquid assets to a concentrated portfolio. If $A \geq L$, then the addition effectively insulates the seller from a liquidity shock, leading to maximum price informativeness (as in Malherbe (2014)). Indeed, the net liquidity shock, $L - A$, is now negative. If instead $A < L$, the addition effectively reduces the liquidity shock to $L - A$; since price informativeness under concentration is independent of the liquidity shock (as long as it is positive), it is unaffected by the new liquid assets. Intuitively, since liquid assets are always fairly priced, selling them to satisfy a shock involves no loss. Upon a shock, if the asset turns out to be good, the seller will sell liquid assets first and only raise $L - A$ from the asset. If the asset is bad, the seller will again sell liquid assets first and raise only $L - A$ from the asset, since raising more would fully reveal the asset as bad.

Now instead consider the effect of adding $A < L$ dollars of assets in a new asset j . To ease the exposition, we will consider the case in which assets i and j are negatively correlated, but the result only requires less than perfect correlation. Upon a shock, if the initial asset i is good (and new asset j is bad), the seller will sell asset j first and thus only partially sell asset i – the same as if the seller instead had liquid assets. The critical difference is if asset i is bad (and new asset j is good). Now, the seller will not sell asset j first. Unlike liquid assets, asset j would suffer an adverse selection discount and so she would suffer a loss by selling it. She instead fully sells asset i . Even though doing so reveals asset i as bad, it is better than selling the higher-quality j .

Put differently, by adding Treasury bills, the seller never has to fully sell asset i , regardless of its quality, since she always sells Treasury bills first. Thus, she always receives a price strictly greater than \underline{v} . However, by adding asset j , the seller may still have to fully sell asset i and receive \underline{v} . Treasury bills provide *uncontingent* liquidity (they are always sold first) but asset j provides *contingent* liquidity (it is not sold first if it is good). Simple intuition might suggest that price informativeness rises with the seller’s liquidity, as it insulates her from a liquidity shock and means that trades are more likely to be driven by information than liquidity needs. However, contingent liquidity depends on an asset’s quality, and so the sale of an asset is more likely to be driven by its quality. In sum, adding liquid assets reduces the net liquidity shock but keeps us within the concentration model. Adding an asset moves us to the diversification model with a moderate shock.

1.4 Discussion of Model Assumptions

The analysis above has shown that diversification increases price informativeness. This section discusses which features of our setting are necessary for this result and which can be relaxed.

Continuum of Assets. The model uses a continuum of assets to invoke the law of large numbers, in turn leading to significant tractability – since we know that the seller will have a proportion τ of good assets, this is the only case that we need to consider. Appendix C.1 shows that the results continue to hold with two assets, albeit with more cases to consider. The intuition is as follows. Under two assets, there are cases in which the assets are either both good or both bad, and so the seller has no trading flexibility. She has trading flexibility – due to owning one good and one bad asset – $2\tau(1 - \tau)$ of the time, rather than all of the time

under a continuum. Flexibility still improves compared to the case of concentration, where she never has trading flexibility because all units are necessarily perfectly correlated with each other. Diversification – whether to a finite number or a continuum of assets – provides trading flexibility since individual shares need not be perfectly correlated. The increment to flexibility is increasing in the number of assets. Specifically, with N assets, the probability that assets are either all good or all bad (and thus there is zero trading flexibility) is $\tau^N + (1 - \tau)^N$, which decreases with N . Our model captures this force by studying the two polar cases, of concentration (no flexibility) and owning a continuum of assets (full flexibility), to highlight the benefits of diversification most clearly.

Buying Additional Units. In the model, the seller has no incentive to buy additional units – and so diversification has no effect on her incentive to do so – because such purchases would be fully revealed as stemming from information. If the seller has the possibility of receiving positive liquidity shocks (e.g. if she were a mutual fund who faces fund inflows), then she can partially disguise an information-based purchase as being motivated by a positive liquidity shock, and so will buy rather than hold good assets. Then, diversification has an additional benefit to price informativeness that is analogous to that in the current model – buying additional shares has greater price impact, since it is less likely to emanate from a liquidity shock.¹³

Noise Traders. In general, informed sellers can make profits on their information for two reasons. First, their trade may be unobservable, because it is pooled with that of noise traders, as modeled by Kyle (1985) in a securities application. Second, their trade may be observable, but the buyer does not know whether it is due to information or a liquidity shock, as modeled by Diamond and Verrecchia (1991) in a securities application. Our model uses the second framework, since it is the fact that liquidity shocks are at the portfolio level that leads to connection between unrelated assets. We conjecture that the results will be robust to adding noise traders to the model. The buyer is now only able to partially infer the probability that a sale comes from the informed seller (rather than noise traders), rather than observing it directly. However, given a probability that the seller has sold asset i , the likelihood that this sale was due to negative information is higher under diversification due to the seller’s flexibility over which assets to sell to satisfy a shock. The model only requires a strictly positive probability of

¹³Note, however, that as long as the seller has the option to hold the inflow as cash, or purchase new assets, rather than being forced to buy more of her existing assets, a purchase of additional units of existing assets holdings will be fully revealing and thus not strictly profitable.

a liquidity shock for diversification to be beneficial (it allows for any $\beta \in (0, 1]$, including $\beta = 1$, i.e. no private information about the liquidity shock), because it is the portfolio-wide liquidity shock that creates the link between trading in the individual assets, and can accommodate any volume of noise trader demand.

Single Buyer. The model assumes that there is a separate buyer for each asset i . This is for two reasons. The first is empirical realism: in reality, there are several market makers for traded securities and different market makers make markets for different securities. Away from a securities application, a conglomerate selling multiple divisions, or a private equity firm selling multiple businesses, will likely be selling them to different buyers, and so the buyer of one division does not know whether another division is up for sale. The second is to highlight the economic forces behind our result – that diversification gives the seller trading flexibility. Our result does not arise because the buyer can observe the seller’s trades in other firms and compare her trade in asset i to that in asset j .

An alternative assumption is to have a single buyer who observes the trades in all assets, such as a market maker for many securities. Appendix C.2 shows that price informativeness can be even higher under diversification than with separate buyers, since the single buyer is able to engage in “relative performance evaluation”. For example, consider the moderate-shock equilibrium of part (ii) of Lemma 2, where (\bar{v}, L) is pooled with $(\underline{v}, 0)$ under separate buyers because both are partially sold. Under a single buyer, prices depend not on the *absolute* trade in a given asset, but the trade *relative* to that in other assets. If other assets are sold more (less), the buyer infers a shock (no shock) and thus that the partially-sold asset is good (bad). Thus, (\bar{v}, L) and $(\underline{v}, 0)$ can now be fully distinguished. By comparing the trade in asset i to that in asset j , the buyer can better discern whether a sale was due to a liquidity shock or low asset value, leading to perfect price informativeness.

Appendix C.2 also shows that there also exists an equilibrium under diversification where prices are fully uninformative, and thus less informative than under concentration. The intuition is as follows. The buyer knows with certainty the value of the seller’s portfolio, which is $\underline{v} + \Delta\tau$ by the law of large numbers. The buyer also observes the seller’s trades across her entire portfolio, and so can always offer a fair price for the portfolio which ensures that he breaks even.¹⁴ If the seller sells a tranche of her entire portfolio (engages in balanced exit), the

¹⁴This contrasts both the case of concentration and the case of diversification with separate buyers, since the buyer for an individual asset does not know whether it is worth \bar{v} or \underline{v} .

buyer knows that it is worth $\underline{v} + \Delta\tau$ and so pays this price. Intuitively, by selling all assets to the same degree, the seller loses on the good assets but gains on the bad assets since the buyer cannot distinguish the two). If the seller engages in imbalanced exit, the buyer knows that assets sold more are bad and worth \underline{v} . The seller thus makes zero profit for all trading strategies (regardless of whether she has suffered a shock) and so is indifferent between them. As a result, there is also an equilibrium in which she retains all assets when $\theta = 0$ and sells all assets when $\theta = L$. Since the seller's trade is independent of asset quality, prices are fully uninformative. Section 3 will show that, when asset values are endogenous, higher price informativeness leads to higher real efficiency. Thus, under the efficiency criterion, the equilibrium with greater price informativeness will be selected. Moreover, Appendix C.1 analyzes the case of two firms, where the law of large numbers does not apply and so the buyer does not know the value of the seller's portfolio, and derives conditions under which price informativeness under a single buyer is higher under any equilibrium under diversification than *any* equilibrium under concentration. Thus, our results are robust to the assumption of a single buyer.

Heterogeneous Assets. Appendix C.3 considers the case in which assets have different valuation distributions, and so information asymmetry Δ and thus the price impact of selling differs across assets. It remains the case that price informativeness is strictly higher under diversification when the shock is small. Regardless of Δ and thus price impact, the seller always receive (weakly) more than \underline{v} by selling a bad asset and less than \bar{v} by selling a good asset, and thus is always better off by selling assets that she knows to be bad and retaining assets she knows to be good. Thus, it remains the case that diversification allows the seller to fully retain good assets upon a small shock, and so a sale fully reveals that an asset is bad. Note that the analysis of holding cash (see discussion at end of Section 1.3) also shows that the model is robust to heterogeneous assets, since cash has information asymmetry of $\Delta = 0$.

Distribution of Liquidity Shocks. While our model assumes a binary liquidity shock $\theta \in \{0, L\}$, our core mechanism, that diversification provides flexibility, applies regardless of the distribution of θ . Even with a more general distribution of liquidity shocks, as long as there is a strictly positive probability that the shock is small, a diversified seller can sometimes retain good assets upon a shock, whereas a concentrated seller is always forced to sell good assets upon a shock.¹⁵

¹⁵Even if there is zero probability that the shock is small, as long as there is a strictly positive probability

Endogenous Portfolio Structure. Our analysis has taken portfolio structure as exogenous and compared the seller’s trading strategy under both concentration and diversification. It thus shows that, when a seller is diversified for any reason (e.g. risk aversion or a single asset being short supply), price informativeness will be higher. If the seller endogenously chose portfolio structure, she would be indifferent between both structures as her expected payoff is the same. Her trading gains when informed equal her trading losses when uninformed, since the only other player in the game is the buyer who breaks even.¹⁶ We will show that the seller will no longer be indifferent under endogenous information or endogenous firm value.

2 Endogenous Information

In the core model, the seller is endowed with private information. This applies to cases in which owning and operating the asset automatically gives the seller information. For example, a conglomerate will have information on the value of its businesses simply by running them. Large shareholders have greater access to firm management, and lenders are able to request information from borrowers at little cost. We now study the case in which the seller now has to acquire private information at a cost. One might think that information acquisition is lower under diversification (thus offsetting our earlier result of greater price informativeness) for two reasons. First, since prices are more revealing of the seller’s information, her ability to profit from it is lower (if unshocked). Second, since the seller owns 1 rather than n units in each asset, she can sell fewer assets upon negative information and so information is less useful to her. Put differently, diversification leads to the investor being spread too thinly and having insufficient skin-in-the-game to motivate information acquisition. We show that, despite these forces, information acquisition may be strictly higher under diversification.

Just after asset values are realized at $t = 1$, for asset i the seller can now pay a cost $c(\lambda_i) \geq 0$ to learn $v_i \in \{\underline{v}, \bar{v}\}$ w.p. $\lambda_i \in [0, 1]$; w.p. $1 - \lambda_i$ she remains uninformed. Whether the seller is informed about asset i is her private information and independent across firms.

that the shock is moderate, diversification still gives the seller flexibility as she will sometimes have flexibility to sell good assets less than bad assets up to a shock.

¹⁶She would not be indifferent if forces well-established in prior research were in operation. For example, if there were noise traders, then the seller’s trading gains when informed are partially at the expense of noise traders rather than only her trading losses when uninformed; then, holding multiple assets may give her access to more noise traders to exploit. Alternatively, differences in liquidity between the assets may affect the ease of establishing her initial position; risk aversion may also lead the seller to prefer diversification.

We assume $c''(\cdot) > 0$ with $c(0) = 0$, $c'(0) = 0$ and $c'(1) = \infty$. We refer to the choice of λ_i as “investigating” or “acquiring information”. Having chosen λ_i , she observes asset values and chooses how much to sell.

As in the baseline model, we let $x_i(v_i, \theta)$ denote the selling strategy for asset i where the asset has expected value $v_i \in \{\underline{v}, \underline{v} + \tau\Delta, \bar{v}\}$ and the seller faces liquidity shock θ . Lemma 3 describes equilibrium investigation under concentration; the selling strategies and prices are in Appendix A.

Lemma 3 (*Information Acquisition, Concentration*): *Consider the model with information acquisition and concentration. In any equilibrium, the seller investigates each asset with intensity $\lambda_{con}^* < 1$, which is unique and defined by the solution to:*

$$c'(\lambda) = \beta(1 - \beta)\tau(1 - \tau)n\Delta \min \left\{ \frac{L/n}{\underline{v}[\lambda(1 - \beta)(1 - \tau) + \beta] + \Delta\beta\tau}, \frac{1}{\lambda(1 - \beta)(1 - \tau) + \beta} \right\}. \quad (11)$$

The intuition behind the investigation threshold, (11), is as follows. Up to a point, the greater the seller’s number of units n , the greater the incentives to investigate. This is because, when her stake n is small, a liquidity shock forces her to sell it in its entirety. Thus, if she learns that the asset is bad, she also sells her entire stake, because doing so disguises her sale as being motivated by a shock. A higher n allows her to sell more units if she learns that the asset is bad, increasing her trading profits, and thus her incentives to gather information. However, after a point, information acquisition becomes independent of n .¹⁷ The seller’s stake n is now sufficiently large that she is no longer forced to sell it in its entirety upon a shock. As a result, she can only partially sell her stake upon learning that the asset is bad, otherwise she is fully revealed, and so further increases in stake size do not increase her investigation incentives.¹⁸

Next, we consider the model under diversification.

Lemma 4 (*Information Acquisition, Diversification*): *Consider the model with information acquisition and diversification.*

¹⁷This point is $\frac{L/n}{\underline{v}[\lambda^*(1 - \beta)(1 - \tau) + \beta] + \Delta\beta\tau} = \frac{1}{\lambda^*(1 - \beta)(1 - \tau) + \beta}$, and information acquisition is now given by $c^{-1}(\lambda^*) = \frac{\beta(1 - \beta)\tau(1 - \tau)\Delta L}{\underline{v}[\lambda^*(1 - \beta)(1 - \tau) + \beta] + \Delta\beta\tau}$.

¹⁸This result also arises in the Edmans (2009).

(i) Suppose $L/n \leq \Lambda(\beta) \cdot (1 - \tau)\underline{v}$ and

$$\Lambda(\beta) \equiv (c')^{-1}(\beta\tau(1 - \tau)\Delta). \quad (12)$$

Then, there exists an equilibrium in which the seller investigates each firm with intensity $\lambda_{div}^* = \frac{L/n}{(1-\tau)\underline{v}}$.

(ii) If $L/n < (1 - \tau)\underline{v}$ and $\beta \in (0, 1]$ then in any equilibrium $\lambda_{div}^* > 0$.

(iii) If $L/n \geq (1 - \tau)\underline{v}$ and $\beta = 1$ then there exists an equilibrium with $\lambda_{div}^* = 0$.

Lemma 3 shows that, since all assets are ex ante identical and the cost function is convex, she chooses a constant $\lambda_i = \lambda^* \forall i$. The intuition behind each part is as follows, starting with part (i). Intuitively, when the seller investigates a certain measure of assets, by the law of large numbers she knows how many will be bad. Thus, when L/n is small, she investigates with just enough intensity to reveal enough bad assets that she can exactly satisfy any liquidity shock by selling them all. She has no incentive to investigate less, since if she suffers a shock, she will have to sell some assets of unknown quality, some of which will be good. She has no incentive to investigate more intensely. Doing so will uncover additional bad assets, but she does not need to sell these assets as she is already satisfying her liquidity needs, and earns no profit by voluntarily selling these assets since selling leads to a price of \underline{v} . The seller's investigation intensity, $\lambda_{div}^* = \frac{L/n}{(1-\tau)\underline{v}}$, is increasing in the per-unit liquidity shock L/n . When the shock is large relative to the number of assets, the seller needs to sell a larger stake in each asset to satisfy a shock. This increases the losses from selling assets of unknown quality that are actually good, and thus the incentives to investigate asset value.

Part (ii) of Lemma 4 shows that seller always investigates with strictly positive intensity when L/n is small, and part (iii) shows that if $\beta = 1$ and L/n is large then there always exists an equilibrium in which the seller does not investigate. Intuitively, if the shock is large and likely, she has little flexibility to keep the good assets and sell the bad ones.

Building on Lemmas 3 and 4, Proposition 2 shows that, if β is sufficiently large and L is sufficiently small, the investigation is higher in *any* equilibrium under diversification than in *any* equilibrium under concentration.¹⁹

¹⁹The seller's incentives to acquire information are determined by the cost per unit of the asset she owns.

Proposition 2 (*Information Acquisition, Comparison of Ownership Structures*): *There is $\bar{\beta} < 1$ such that, if $\beta > \bar{\beta}$ and $L/n \leq \Lambda(\bar{\beta}) \cdot (1 - \tau)\underline{v}$, the seller acquires strictly more information in any equilibrium under diversification than in any equilibrium under concentration.*

The intuition is as follows. If the seller suffers a liquidity shock, information has no value under concentration as she has to sell the same number of units regardless of whether the asset is good or bad. In contrast, information has value under diversification since it guides her on which assets to sell to satisfy the shock. Thus, investigation incentives are higher under diversification when the probability of the shock β is sufficiently high. Investigation incentives are increasing with L/n under both concentration and diversification due to the earlier intuitions. However, they rise less under diversification. Information on asset quality is only valuable under diversification if the shock is sufficiently small that the seller can satisfy it by selling only bad assets. Thus, investigation incentives are higher when the shock is sufficiently small.

Note that if the seller endogenously chose her initial position, she would prefer the ownership structure under which information acquisition incentives are *lower*. This is because information is a deadweight cost to her. While information increases her trading gains if she does not suffer a shock, and thus can trade freely on her information, it also leads to a lower sale price and thus increases her trading losses if shocked. Information also does not increase the ex ante value of her portfolio since asset values are exogenous. The seller cannot commit not to acquiring information, and so a rational buyer sets a pricing function taking into account his expectation of the seller's level of information.

Proposition 3 shows that price informativeness can be higher under *any* diversification equilibrium than under *any* concentration equilibrium, when information acquisition is endogenous. Moreover, even if information acquisition is lower under diversification, price informativeness may be higher due to the results of Section 1 – low information of which a high proportion is incorporated in prices may dominate high information of which a low proportion is incorporated

While the aggregate cost of acquiring information across all assets in the portfolio is sizable, note that the cost of acquiring information per asset is infinitesimal on an absolute scale when the investor owns a continuum of assets. The assumption of a continuum of assets is made for tractability. In Appendix C.1, we show that the core result of Section 1 holds even if the seller owns only two assets under diversification. This section's result on information acquisition also likely hold with two assets. Indeed, as we show in Lemma 3, if $\beta = 1$ then $\lambda_{con}^* = 0$ under concentration. Under diversification to two assets, if $\beta = 1$ and L is sufficiently small that the seller can meet her liquidity needs by selling only one asset, then $\lambda_{div}^* > 0$ in any equilibrium. To see why, note that if on the contrary $\lambda_{div}^* = 0$, the seller has strict incentives to acquire some information on both assets (since $c(0) = c'(0) = 0$) and use it to sell the bad asset and keep the good one.

in prices.

Proposition 3 (*Price informativeness Under Information Acquisition*):

- (i) *There is $\beta^* < 1$ such that if $\beta > \beta^*$ and $L/n \leq \Lambda(\beta^*) \cdot (1 - \tau)\underline{v}$ then for any equilibrium under diversification and any equilibrium under concentration*

$$P_{div,info}(\bar{v}, \tau) > P_{con,info}(\bar{v}, \tau) \text{ and } P_{div,info}(\underline{v}, \tau) \leq P_{con,info}(\underline{v}, \tau). \quad (13)$$

Moreover, there is an equilibrium under diversification such that the above inequalities are strict with respect to any equilibrium under concentration.

- (ii) *There is $\beta^{**} < 1$ and $n^{**} < \infty$ such that if $\beta > \beta^{**}$, $n > n^{**}$, and $L/n \leq \Lambda(\beta^{**}) \cdot (1 - \tau)\underline{v}$, there is an equilibrium under diversification and an equilibrium under concentration such that the seller acquires strictly more information under concentration yet the inequalities in (13) hold strictly.*

3 Endogenous Asset Value

The core model has shown that a diversified portfolio improves price informativeness. We now demonstrate the real effects of this result by endogenizing asset value as depending on a real action. We revert to the case of exogenous private information for simplicity, but the results continue to hold for endogenous information as long as price informativeness rises with diversification, i.e. the conditions in Proposition 3 are satisfied.

Let the asset now be a security in a firm: debt, equity, or any security monotonic in firm value. The seller can thus be interpreted as an institutional investor such as a hedge fund, mutual fund, or bank. There are a total of $m \geq n$ securities in the firm, of which z are owned by the seller (as before) and the remaining $m - z$ are owned by dispersed investors (households) who play no role. Each firm is run by a separate manager, who takes action $a_i \in \{0, 1\}$ at $t = 1$. When $a_i = 1$ (0), the value of each asset is $v_i = \bar{v}$ (\underline{v}). Examples of $a_i = 0$ include shirking, cash flow diversion, perk consumption, and empire building. We refer to $a_i = 0$ as “shirking” and $a_i = 1$ as “working.” A good (bad) firm is one in which the manager has worked (shirked). Action $a_i = 1$ imposes a cost $\tilde{c}_i \in [0, \infty)$ on manager i , privately observed by the

manager prior to deciding his action. The probability density function of \tilde{c}_i is given by f and its cumulative distribution function by F . Both are continuous and have full support. We assume \tilde{c}_i are i.i.d. across firms, and that $E[\tilde{c}_i] \leq \Delta$, so that $a_i = 1$ is ex-ante efficient.

Manager i 's objective function is given by:

$$u_{M,i} = (1 - \omega) v(a_i) + \omega p_i - \tilde{c}_i a_i. \quad (14)$$

The manager cares about firm value and also the $t = 2$ security price; these price concerns are captured by ω . If the security is equity, ω refers to stock price concerns, which are standard in such models and can stem from a number of sources introduced in prior work. Examples include takeover threat (Stein (1988)), concern for managerial reputation (Narayanan (1985), Scharfstein and Stein (1990)), or the manager expecting to sell his own securities at $t = 2$ (Stein (1989)). To our knowledge, exit theories have not previously considered the potential application to debt securities. The manager may care about the short-term debt price, or the firm's reputation in debt markets, as it will affect the ease at which he can raise additional debt (e.g. Diamond (1989)).

This application is thus a model of “governance through exit” (Admati and Pfleiderer (2009), Edmans (2009)). The seller exerts governance by selling the asset if the manager shirks. Doing so reduces the asset price and punishes the manager ex post; the threat of exit increases his incentives to work ex ante. However, the punishment for shirking depends not on the decision to sell per se but the price impact of the sale – and hence whether the seller is concentrated or diversified. We thus extend exit models to the case of multiple firms and show how the strength of exit depends on the seller's holdings of potentially unrelated firms.

As in the core model, the seller privately observes v_i under concentration and $\mathbf{v} \equiv [v_i]_{i=0}^n$ under diversification, but neither she nor the buyers observe $\tilde{\mathbf{c}} \equiv [\tilde{c}_i]_{i=0}^n$. As before, her utility is given by (1) under concentration and (2) under diversification. We will abuse language slightly by using the phrase “the manager will be sold” to refer to the securities of the firm run by the manager being sold. We continue to focus on symmetric equilibria, in which the managers follow the same strategy and each buyer uses a symmetric pricing function. The equilibrium concept is as in Section 1 with the following additions: (vii) a decision rule by each manager i that maximizes his expected utility $u_{M,i}$ given his information on \tilde{c}_i , other managers' strategies, the buyer's price-setting rule, and the seller's trading strategy, and (viii) each buyer

forms expectations about τ that are consistent with (vii).

We first derive the following threshold rule that holds under both concentration and diversification:

Lemma 5 *In any equilibrium and under any ownership structure, there is a c^* such that manager i chooses $a_i = 1$ if and only if $\tilde{c}_i \leq c^*$.*

Manager i works only if his weight on the value gain $(1 - \omega) \Delta$ plus ω times the expected price rise exceeds his cost. The maximum price rise is Δ , which arises if the price is fully informative. Thus, in any equilibrium, $c^* \leq (1 - \omega) \Delta + \omega \Delta = \Delta$. Ex-ante total surplus (firm value minus the cost of effort) in equilibrium is increasing in c^* if and only if $c^* \leq \Delta$. Since the manager always chooses $c^* \leq \Delta$, a higher c^* always increases total surplus. We thus define efficiency as the maximization of c^* .

We defer the analysis of the equilibria under a concentrated and diversified portfolio (the analogs of Lemmas 1 and 2) to Appendix B (Lemmas 6 and 7) and move straight to comparing the most efficient equilibria, which is given in Proposition 4 below (Appendix B analyzes other equilibria, and also contains the proofs for the results in this section.)

Proposition 4 *(Comparison of most efficient equilibria, endogenous asset values): The working threshold under the most efficient equilibrium, $c_{div,end}^{**}$, is strictly higher than under concentration if $L/n < \underline{v}$ and the same if $L/n \geq \underline{v}$.*

Proposition 4 states that, for a large shock ($L/n > \underline{v}$), governance is the same under both ownership structures, because prices and trading strategies are the same. For a small or moderate shock ($L/n < \underline{v}$), governance is strictly superior under the most efficient equilibrium under diversification than under concentration. This is for two reasons. First, diversification increases the punishment for shirking. Under concentration, exit is consistent with the investor suffering a liquidity shock and so a sold firm receives a relatively high price. Thus, the punishment for shirking is low. Under diversification and a small shock, exit is fully revealing of shirking and leads to the lowest possible price of \underline{v} . Under diversification and a moderate shock, a bad firm is fully sold and thus fully revealed. Intuitively, diversification creates a tournament between the n managers, who know that the seller observes their efforts and will sell the worst performers. Since the market anticipates that the worst performers are sold, this amplifies the

disciplinary power of exit. Moreover, this tournament means that, even if there is no explicit relative performance evaluation on the managers' contracts, they will be evaluated according to relative performance by the seller. Second, diversification increases the reward for working. Under concentration, a good firm is automatically sold under a shock, and so the reward for working is low. Under diversification, it is retained upon a small shock and only partially sold upon a moderate shock.

If the seller can choose the structure of her portfolio, she will choose the structure that maximizes governance (i.e. a diversified portfolio, under the most efficient equilibrium) as long as the purchase of her initial position is not fully observed (as in, e.g., Kyle and Vila (1991)). This is because she can acquire assets at less than their fundamental value, and so she shares in the value created by improved governance. If her trade is fully observed, she has to acquire her initial position at their full value, including any governance benefits, and so would be indifferent. The seller may choose to be diversified for reasons outside the model, e.g. risk reduction concerns, "prudent man" rules, or downward-sloping demand curves for a single asset. In this case, our results suggest that diversification for private risk reduction or price impact reasons can have a social benefit by improving governance.

4 Additional Applications

This section discusses additional applications of our core result on the link between portfolio diversification and price informativeness.

Governance Through Voice. Section 3 endogenized asset value as depending upon an action taken by the manager, as in models of governance through exit. Alternatively, the asset value could depend on a monitoring action taken by an institutional investor, as in models of governance through voice. Faure-Grimaud and Gromb (2004) consider a model in which the seller only trades (as in our core model) and the monitoring action is undertaken by a separate institution who is also concerned about the $t = 2$ stock price. This model is almost identical to the model of Section 3, with the monitor replacing the manager. Diversification increases $t = 2$ price informativeness and thus the reward for monitoring.

In Kahn and Winton (1998) and Maug (1998), the monitor is the same as the seller, e.g. the seller is a blockholder who engages in intervention. This leads to a similar model to Section

3 except that the action is taken by the seller. The results are available upon request and we discuss the intuition here, which mirrors that of Section 3. Under concentration, the seller’s incentives to monitor are reduced for two reasons. First, if she monitors and increases the asset value to \bar{v} , she may suffer a liquidity shock and be forced to sell for a price below \bar{v} , reducing the payoff to monitoring. Second, if she does not monitor, she can sell some assets (“cut and run”), and pretend that the sale is of a good asset but motivated by a liquidity shock. This increases the payoff to not monitoring. The same two reasons mean that diversification weakly increases the seller’s incentive to monitor. With a small shock, the seller never needs to sell a monitored firm. With a moderate shock, the seller is forced to sell a monitored firm but only partially, and so receives a higher price than under concentration. In addition, the payoff to cutting and running is now lower since prices are more informative. A sale is more indicative that the seller has not monitored, since if she had monitored and suffered a liquidity shock, she would have sold other firms instead. With a large shock, diversification does not provide flexibility and so this benefit is absent. The positive effect of flexibility (under a small or moderate shock) on monitoring incentives must be weighed against the fact that the seller now only has 1 rather than n units in each firm, which reduces her incentive to monitor.

Overall, the exit and voice applications suggest that diversification can strengthen governance – if the greater price informativeness outweighs any potential loss from the investor being spread too thinly (under the exit application, information acquisition may be lower although need not be; under the voice application, the investor enjoys a smaller share of the gains from monitoring under diversification). This is more likely to be the case if the liquidity shock is small, so that the increase in price informativeness is higher, and the number of firms is small, so that the effect of being spread too thinly is low. This application thus has the potential to justify why shareholders own blocks in multiple firms and banks lend large amounts to multiple borrowers, despite the free-rider problem. Existing justifications are typically based on diversification of risk. While conventional wisdom might suggest that the diversification induced by risk concerns necessarily weakens governance, our model highlights an opposing force. Indeed, Kang, Luo, and Na (2017) find that institutional investors are more effective at governance the more blocks they have in other companies, controlling for portfolio size. This superior governance may arise due to greater price informativeness as in our paper, or other channels such as additional blocks leading to learning-by-doing. Similarly, our model suggests that mergers between investors, which do not reduce an investor’s stake in a given firm and

thus do not spread her too thinly, should improve governance, even if the investors do not have common holdings and so the merger does not increase their stake in a given firm. A more minor contribution is that our model extends exit models to governance by debtholders as well as equityholders.

Relatedly, while existing studies typically use the size of the largest blockholder or the number of blockholders as a measure of governance, our paper theoretically motivates a new measure – the number of other large stakes owned by its main shareholder or creditor, as studied by Kang, Luo, and Na (2017). Faccio, Marchica, and Mura (2011) empirically study a related measure, the concentration of an asset in a seller’s portfolio. They argue that diversification can be desirable because a concentrated seller will turn down risky, positive-NPV projects, unlike our channel.

Conglomerates. Relatedly, the model can be applied to a corporate headquarters’ or private equity general partners’ decision to diversify into multiple uncorrelated business lines rather than be focused in a single business or have multiple correlated businesses. Our results suggest that diversified firms face a more severe adverse selection problem when divesting than concentrated firms, since it is harder to justify a divestment as resulting from a liquidity shock.²⁰ Stein (1997) shows that an advantage of conglomeration is “winner-picking” – the headquarters can invest surplus funds into the business that has the best investment opportunities at the time. One may think that a related advantage is “loser-picking” – if it suffers a liquidity shock, it can choose to sell the most poorly-performing business. However, potential buyers know this and so the headquarters face a more, not less, adverse selection problem when selling.²¹

On the other hand, this more severe adverse selection problem may strengthen incentives to improve business value. Under the exit application, it is the divisional manager whose actions

²⁰Where the asset is a business, it may also be sold for strategic reasons such as dissynergies. However, evidence suggests that liquidity needs are an important motive for asset sales (e.g., Borisova, John, and Salotti (2013), Campello, Graham, and Harvey (2010)). Our analysis studies how diversification affects the likelihood that a sale is driven by a liquidity shock rather than private information, effectively holding synergy motives constant when comparing diversification to concentration.

²¹In our model, the seller only cares about the $t = 3$ fundamental value of her portfolio. Edmans and Mann (2017) show that, if she also cares about the $t = 2$ market value of her portfolio (e.g. is an investor who faces interim fund flows), divesting may be less attractive in a single-segment firm because the low price received for sold plants implies a low market value for the retained plants, since they are perfectly correlated. Under diversification, the sold divisions need not be perfectly correlated with the retained ones, and so the low price on the former need not imply a high price for the latter. That model features exogenous asset values, a publicly-known liquidity shock and only consider the case of diversified and not concentrated ownership.

affect firm value, and his labor market reputation may be affected by the market's perceived value of his division. If a multi-segment firm sells a division, this is a sign that the division is poorly performing. If a single-segment firm sells some plants, this may be because the headquarters has suffered a liquidity shock. Under the voice application, it is the headquarters whose actions affect firm value, and its incentives to monitor are greater under diversification.

A similar result holds in the case in which the headquarters' decision is not to sell a business, but instead to shut it down. In this case, there is no sale price, and the headquarters' payoff from shut down is its alternative use of capital, which is independent of the market's perception of the shut-down business. Thus, the seller is not concerned with price impact or adverse selection. Appendix C.4 shows that, even with a fixed reservation payoff, price informativeness is always weakly higher under diversification, because it remains the case that the seller does not need to sell good assets upon a liquidity shock. Again, this greater price informativeness will improve governance through either exit or voice. This model can also apply to other discontinuation decisions, such as a bank ceasing to lend or a venture capitalist not investing in a future financing round.²² The threat of being the only business with which the bank or venture capitalist terminates the relationship improves the manager's effort incentives.

Gervais, Lynch, and Musto (2005) show that mutual fund families can add value by monitoring multiple managers, since firing one manager increases sellers' perceived skill of retained managers.²³ Inderst, Mueller, and Münnich (2007) show that when a seller finances several entrepreneurs, an individual entrepreneur may exert greater effort. To obtain refinancing, he needs to deliver not only good absolute performance, but also good performance relative to his peers. In Fulghieri and Sevilir (2009), multiple entrepreneurs compete for the limited human capital of a single venture capitalist. These effects are similar to the relative performance evaluation channel under a single buyer. However, these papers do not consider the case of our core model where trades in other assets are unobservable and so there is no relative performance evaluation, nor is there an analog of the liquidity shock.

Learning. In Section 3, price informativeness affects real asset values by affecting managers' incentives to take real decisions. The survey of Bond, Edmans, and Goldstein (2012) discusses

²²In the model, the discontinuation decision has no direct effect on firm value, for example because there are other banks or venture capitalists who can provide financing. This highlights the channel through which the trading / discontinuation decision affects firm value – indirectly through affecting incentives.

²³In addition, Gervais, Lynch, and Musto (2005) assume that the fund family can commit to a firing strategy; in our model, the seller makes the trading decision that maximizes her ex post profits.

a second channel through which price informativeness can have real effects: learning. In learning models, asset value now depends on an action taken by a decision maker, who infers information from the asset price to guide his action. For example, a manager could learn investment opportunities from the stock price to decide whether to expand or disinvest; a board could learn management quality to decide whether to fire the manager; a policymaker could learn firm quality to decide whether to intervene in or bail out a company; or customers, suppliers, employees, and capital providers could learn firm quality to decide whether to initiate, continue, or terminate their relationship with the firm. Diversified investors may reveal more information into prices, thus allowing decision makers to glean more information from prices and in turn increasing real efficiency.

5 Conclusion

This paper has shown that the informativeness of the price of an asset depends on an informed seller's holdings of other assets, even if they are unrelated and even if the buyer cannot observe her trades in those assets. A diversified investor has the choice of which assets to sell upon a liquidity shock. She cannot commit not to selling the worst assets first, and so an asset sale is more revealing of low asset quality than a liquidity shock. Thus, her trades convey more information, increasing price informativeness. This result has implications outside a trading context. Examples include a director's decision to quit a board, an asset's decision to exit or scale back a line of business, or an employer's decision to fire a worker. In all of these cases, the negative inference resulting from termination is attenuated is stronger if the decision-maker had many other relationships that she could have terminated instead.

Importantly, diversifying by adding risky assets to the seller's portfolio is critically different to adding financial slack such as Treasury bills, because they provide only contingent rather than uncontingent liquidity. Moreover, even though diversification increases the extent to which private information is revealed in prices and reduces the seller's skin-in-the-game in any individual asset, it may raise information acquisition incentives. Since diversification gives her the option to sell bad assets and retain good ones under a liquidity shock, it increases her incentives to learn asset quality.

We show that the greater price informativeness increases the incentives to improve asset value when it is endogenous. If a manager works, he is more likely to be retained since the

seller has other assets that she can sell upon a shock. If a manager shirks and is sold, this leads to a lower stock price. This result suggests that concentrating ownership of many firms within a small number of investors may strengthen governance. Thus, mergers of investors and demergers of firms may improve governance; demergers of investors and mergers of firms reduce it. Similarly, diversification by a corporate headquarters, private equity general partner, venture capitalist, or bank can improve incentives of firm managers.

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A Proofs

Proof of Lemma 1. Let $x^*(v, \theta)$ be an equilibrium strategy for type- (v, θ) . We start by proving that there is a unique $\bar{x} > 0$ such that $x^*(\underline{v}, L) = x^*(\underline{v}, 0) = x^*(\bar{v}, L) = \bar{x} > x^*(\bar{v}, L)$. We argue five points:

1. *In any equilibrium, $x^*(\underline{v}, L) > 0$, $x^*(\underline{v}, 0) > 0$, and $x^*(\bar{v}, L) > 0$.* Proof: Suppose on the contrary that $x^*(\bar{v}, L) = 0$. Since $\beta > 0$, the seller can raise a positive amount of revenue (and thereby strictly increasing her payoff), by deviating to selling n units, a contradiction. Note that, $p(x^*(\bar{v}, L)) > \underline{v}$. Next, suppose on the contrary $x^*(\underline{v}, \theta) = 0$. The seller's payoff is \underline{v} . However, since $x^*(\bar{v}, L) > 0$ and $p(x^*(\bar{v}, L)) > \underline{v}$, the seller obtains a strictly higher payoff than \underline{v} by deviating to $x^*(\bar{v}, L)$, a contradiction.
2. *In any equilibrium, $x^*(\bar{v}, 0) \neq x^*(\underline{v}, 0)$ and $x^*(\bar{v}, 0) \neq x^*(\underline{v}, L)$.* Proof: If on the contrary $x^*(\bar{v}, 0) = x^*(\underline{v}, \theta)$, then $p(x^*(\bar{v}, 0)) < \bar{v}$. Based on point 1, $x^*(\underline{v}, \theta) > 0$. Therefore, the payoff of type $x^*(\bar{v}, 0)$ is strictly smaller than \bar{v} . This creates a contradiction, since type $x^*(\bar{v}, 0)$ can obtain a payoff of \bar{v} by fully retaining the asset.
3. *In any equilibrium, $x^*(\bar{v}, 0) \neq x^*(\bar{v}, L)$.* Proof: Suppose not. Based on point 1, $x^*(\bar{v}, 0) > 0$. Based on point 2, $p(x^*(\bar{v}, 0)) = \bar{v}$. Moreover, type- (\bar{v}, L) can either satisfy her liquidity need by selling $x^*(\bar{v}, 0)$ from each asset, or she obtains the highest revenue possible by following strategy $x^*(\bar{v}, 0)$. Moreover, $p(x^*(\underline{v}, \theta)) = \underline{v}$, and so type- (\underline{v}, θ) receives a payoff of \underline{v} . However, type- (\underline{v}, θ) can obtain a strictly higher payoff by deviating to $x^*(\bar{v}, 0) > 0$, a contradiction.
4. *In any equilibrium, there is $\bar{x} > 0$ such that $x^*(\underline{v}, L) = x^*(\underline{v}, 0) = x^*(\bar{v}, L) = \bar{x}$.* Proof: Let $x^*(\bar{v}, L) = \bar{x}$, and note that based on point 1, $\bar{x} > 0$. Moreover, the seller can either satisfy her liquidity need by selling \bar{x} from each asset, or she obtains the highest revenue possible by following strategy \bar{x} . Suppose on the contrary, $x^*(\underline{v}, \theta) \neq \bar{x}$ for some $\theta \in \{0, L\}$. Then, based on point 2, it must be $p(x^*(\underline{v}, \theta)) = \underline{v}$, and the payoff of type (\underline{v}, θ) is \underline{v} . However, this creates a contradiction since type (\underline{v}, θ) can deviate to \bar{x} and generate a payoff strictly higher than \underline{v} and meet her liquidity needs.
5. *In any equilibrium, $x^*(\bar{v}, 0) < \bar{x}$.* Proof: from points 2 and 3, $x^*(\bar{v}, 0) \neq \bar{x}$, and hence, $p(x^*(\bar{v}, 0)) = \bar{v}$. Suppose on the contrary $x^*(\bar{v}, 0) > \bar{x}$. Then, type $(\underline{v}, 0)$ has a strictly profitable deviation from \bar{x} to $x^*(\bar{v}, 0)$: she can sell strictly more units from a bad asset at a strictly higher price, a contradiction.

Given the claims above, Bayes' rule implies $p_i(\bar{x}) = \bar{p}_{con}(\tau)$, as given by the lemma. We prove that in any equilibrium $\bar{x} \leq \bar{x}_{con}(\tau)$. Suppose on the contrary that $\bar{x} > \bar{x}_{con}(\tau)$. Then it has to be $\bar{x}_{con}(\tau) = \frac{L}{\bar{p}_{con}(\tau)} < n$, and therefore, $\bar{x}\bar{p}_{con}(\tau) > L$. Since the price function is non-increasing, there is $\varepsilon > 0$ such that $(\bar{x} - \varepsilon)p(\bar{x} - \varepsilon) \geq L/n$. This implies that type (\bar{v}, L) will strictly prefer deviating to $\bar{x} - \varepsilon$, a contradiction. We conclude $\bar{x} \leq \bar{x}_{con}(\tau)$. Next, we prove that if $L/n \leq \underline{v}$ then $\bar{x} = \bar{x}_{con}(\tau)$ in any equilibrium. Suppose on the contrary that $\bar{x} < \bar{x}_{con}(\tau)$. This implies that the seller does not raise L in equilibrium by selling \bar{x} . Consider a deviation where the seller sells all units. Since $p(n) \geq \underline{v}$, the revenue raised would be at least $n\underline{v} \geq L$, and therefore, the deviation is optimal, a contradiction. Suppose $L/n > \underline{v}$. In this case, any $\frac{n\underline{v}}{\bar{p}_{con}(\tau)} \leq \bar{x} \leq \bar{x}_{con}(\tau)$ can be an equilibrium. Indeed, showing that $\bar{x} < \frac{n\underline{v}}{\bar{p}_{con}(\tau)}$ can be done by noting that there is an optimal deviation to selling n units. Among all $\frac{n\underline{v}}{\bar{p}_{con}(\tau)} \leq \bar{x} \leq \bar{x}_{con}(\tau)$, we select the equilibrium that satisfies the seller's liquidity shock, which is $\bar{x}_{con}(\tau)$.

Next, note that in equilibrium it must be $x^*(\bar{v}, 0) > 0 \Rightarrow p(x^*(\bar{v}, 0)) = \bar{v}$. Since $x^*(\bar{v}, 0) = 0$, the price function given by (4) is consistent with (3) and is non-increasing. Note that (3) is incentive compatible given (4). First, the equilibrium payoff of type $(\bar{v}, 0)$ is \bar{v} , the highest possible. Second, since $\bar{p}_{con}(\tau)\bar{x}_{con}(\tau) \leq L/n$ and $p^*(x)$ is flat on $(0, \bar{x}_{con}]$, deviating to $(0, \bar{x}_{con}]$ generates revenue strictly lower than L , and so is suboptimal if $\theta = L$. Moreover, since $x > \bar{x}_{con}(\tau) \Rightarrow p^*(x) = \underline{v}$, the seller has no optimal deviation to $x > \bar{x}_{con}(\tau)$, regardless of asset value. Last, it is easy to see that $x = \bar{x}_{con}(\tau)$ is optimal for type $(\underline{v}, 0)$. ■

Proof of Lemma 2. Suppose $L/n \leq \underline{v}(1 - \tau)$. The seller can raise at least L by selling only bad assets, even if she receives the lowest possible price of \underline{v} . Since the seller is never forced to sell a good asset, she sells a positive amount $x'_i > 0$ from a good asset only if $p(x'_i) = \bar{v}$, i.e. she does not sell x'_i from a bad asset. We first argue that, in any equilibrium, $x_i > 0 \Rightarrow p(x_i) < \bar{v}$. Suppose on the contrary there is $x'_i > 0$ s.t. $p(x'_i) = \bar{v}$, and let x'_i be the highest quantity with this property. The seller chooses not to sell x'_i from a bad asset only if there is x''_i that she chooses with strictly positive probability, where

$$x''_i p_i(x''_i) + (1 - x''_i) \underline{v} \geq x'_i p_i(x'_i) + (1 - x'_i) \underline{v}. \quad (15)$$

The above inequality requires $p_i(x''_i) > \underline{v}$. Since she sells x''_i from a bad asset with positive probability, we have $p_i(x''_i) < \bar{v}$. Given this price, she will never sell x''_i from a good asset, contradicting $p_i(x''_i) > \underline{v}$. Therefore, she sells x'_i from a bad asset with strictly positive probability, which contradicts $p(x'_i) = \bar{v}$. We conclude that in any equilibrium $x_i > 0 \Rightarrow p(x_i) < \bar{v}$,

and so $v_i = \bar{v} \Rightarrow x_i = 0$. Note that the condition on $\underline{x}(\theta)$ simply requires that the seller sells enough of the bad assets to meet her liquidity needs, given by the realization of θ . Last, $p^*(0)$ follows from Bayes' rule and the observation that $v_i = \bar{v} \Rightarrow x_i = 0$. This completes part (i).

Next, suppose $\underline{v}(1 - \tau) < L/n$. We proceed by proving the following claims.

1. In any equilibrium there is a unique $\bar{x} > 0$ s.t. $x_i^*(\bar{v}, L) = x_i^*(\underline{v}, 0) = \bar{x}$. To prove this, let $\bar{x} = x_i^*(\bar{v}, L)$. Since $\underline{v}(1 - \tau) < L/n$, it must be $\bar{x} > 0$. We denote $p_i(\bar{x}) = \bar{p}$. Since the seller sells \bar{x} of a good asset, $\bar{p} > \underline{v}$. We argue that, in any equilibrium, if $\theta = 0$ then she sells \bar{x} of every bad asset. Suppose she sells a different quantity. Recall that $p_i(x_i^*(\bar{v}, 0)) = \bar{v}$ implies that she does not sell $x_i^*(\bar{v}, 0)$ of a bad asset in equilibrium. Since $x_i^*(\underline{v}, 0) \neq \bar{x}$ and $x_i^*(\underline{v}, 0) \neq x_i^*(\bar{v}, 0)$, we must have $p_i(x_i^*(\underline{v}, 0)) = \underline{v}$, which yields a payoff of \underline{v} . This creates a contradiction since she has strict incentives to deviate and sell \bar{x} of a bad asset, thereby obtaining a payoff above \underline{v} . Note that this implies that $\bar{p} < \bar{v}$.
2. In any equilibrium, either $x_i^*(\underline{v}, L) = \bar{x}$ w.p. 1, or $x_i^*(\underline{v}, L) = 1$ w.p. 1, where \bar{x} is defined as in Claim 1. To prove this, note that the seller cannot sell $x_i^*(\bar{v}, 0)$ of a bad asset in equilibrium. Therefore, if $x_i^*(\underline{v}, L) \neq \bar{x}$, then $p_i(x_i^*(\underline{v}, L)) = \underline{v}$. Suppose $x_i^*(\underline{v}, L) \neq \bar{x}$ and $x_i^*(\underline{v}, L) < 1$. Then, she can always deviate to fully selling a bad asset, and not selling some good assets, to keep revenue constant. Her payoff from selling a bad asset is no lower (since she previously received \underline{v} for each bad asset), but by not selling some good assets, for which she previously received $\bar{x}\bar{p} + (1 - \bar{x})\bar{v} < \bar{v}$, she increases her payoff. Therefore, $x_i^*(\underline{v}, L) \in \{\bar{x}, 1\}$, as required.
3. If in equilibrium $x_i^*(\underline{v}, L) = 1$ and $\bar{x} < 1$ then $L/n < \underline{v}$ and $\bar{x} = \bar{x}_{div}(\tau)$, as given by (7). To prove this, since $x_i^*(\underline{v}, L) = 1$ and $v_i = \bar{v} \Rightarrow x_i^* < 1$, $p_i(1) = \underline{v}$. Moreover, given claims 1 and 2, and by Bayes' rule, \bar{p} is given by $\bar{p}_{div}(\tau)$, as given by (8). Suppose $\theta = L$. Since $\bar{p}_{div}(\tau) > \underline{v}$, the seller chooses $x_i^*(\underline{v}, L) = 1$ only if the revenue from selling \bar{x} from all assets is strictly smaller than L and also the revenue from selling \bar{x} of all good assets and 1 from all bad assets, i.e.

$$\bar{x}\bar{p}_{div}(\tau) < \min\{(1 - \tau)\underline{v} + \tau\bar{x}\bar{p}_{div}(\tau), L/n\} \Leftrightarrow \bar{x}\bar{p}_{div}(\tau) < \min\{\underline{v}, L/n\}.$$

Intuitively, we require $\bar{x}\bar{p}_{div}(\tau) < \underline{v}$, since the seller receives $\bar{x}\bar{p}_{div}(\tau)$ by partially selling \bar{x} of a bad asset for price $\bar{p}_{div}(\tau)$, and \underline{v} by fully selling a bad asset for price \underline{v} . In

equilibrium, she would only fully sell a bad asset if doing so raises more revenue.

We now prove that $(1 - \tau)\underline{v} + \tau\bar{x}\bar{p}_{div}(\tau) = L/n$, i.e. fully selling bad assets and selling \bar{x} of good assets raises exactly L . We do so in two steps. We first argue that this strategy cannot raise more than L , i.e.

$$(1 - \tau)\underline{v} + \tau\bar{x}\bar{p}_{div}(\tau) \leq L/n. \quad (16)$$

Suppose not. Then, the seller has “slack”: she can deviate by selling only $\bar{x} - \varepsilon$ instead of \bar{x} from each good asset, while still meeting her liquidity need. Since we consider non-increasing price functions, $p_i(\bar{x} - \varepsilon) \geq \bar{p}_{div}(\tau)$, and so for small $\varepsilon > 0$, she still raises at least L . Her payoff is strictly higher since she sells less from the good assets. We next argue that this strategy cannot raise less than L , i.e.

$$(1 - \tau)\underline{v} + \tau\bar{x}\bar{p}_{div}(\tau) \geq L/n. \quad (17)$$

Suppose not. If the strategy did not raise L , then it must be that $\underline{v} \leq (1 - \tau)\underline{v} + \tau\bar{x}\bar{p}_{div}(\tau)$, i.e. the alternative strategy of fully selling her entire portfolio raises even less revenue. Therefore, $\underline{v} \leq \bar{x}\bar{p}_{div}(\tau)$, which contradicts $\bar{x}\bar{p}_{div}(\tau) < \underline{v}$. Intuitively, if fully selling an asset for \underline{v} raises less revenue than selling \bar{x} of an asset for $\bar{p}_{div}(\tau)$, then the seller would not pursue the strategy of fully selling bad assets. Combining (16) and (17) yields

$$(1 - \tau)\underline{v} + \tau\bar{x}\bar{p}_{div}(\tau) = L/n$$

as required, implying $\bar{x} = \bar{x}_{div}(\tau)$, and $\bar{x}_{div}\bar{p}_{div}(\tau) < \underline{v}$ implies $L/n < \underline{v}$ as required.

4. If in equilibrium $x_i^*(\underline{v}, L) = \bar{x}$ then $\underline{v}\frac{1-\tau}{\beta\tau+1-\tau} \leq L/n$, $\bar{p} = \bar{p}_{con}(\tau)$ and $\bar{x} = \bar{x}_{con}/n$. To prove this, since prices are non-increasing, we must have $\bar{x}\bar{p} \leq L/n$. Otherwise, if $\theta = L$ the seller deviates by selling $\bar{x} - \varepsilon$ instead of \bar{x} from a good asset. For small $\varepsilon > 0$, she can raise the same amount of revenue and sell less from the good assets. Note that $x_i^*(\underline{v}, L) = \bar{x} \Rightarrow \bar{p} = \bar{p}_{con}(\tau)$. Suppose on the contrary that such an equilibrium exists and

$$L/n < \underline{v}\frac{1-\tau}{\beta\tau+1-\tau}.$$

We argue that there is an optimal deviation to fully selling all bad assets, and selling

x' from good assets, for some $x' \in (0, \bar{x}]$. Since $\frac{L/n}{\underline{v}} < \frac{1-\tau}{\beta\tau+1-\tau} < 1$, she can always raise at least L by selling all assets. Therefore, it must be $\bar{x}\bar{p}_{con}(\tau) = L/n$. Moreover, $\bar{p}_{con}(\tau) > \underline{v} \Rightarrow \bar{x} < 1$. Since \bar{x} is an equilibrium, $xp(x) < L/n$ for any $x < \bar{x}$. Let

$$x' = \frac{L/n - (1-\tau)\underline{v}}{\tau\bar{p}_{con}(\tau)}$$

Note that $L/n - (1-\tau)\underline{v} > 0$ implies $x' > 0$ and $\bar{x}\bar{p}_{con}(\tau) = L/n < \underline{v}$ implies $x' < \bar{x}$. By deviating to fully selling all bad assets and selling only $x' \leq \bar{x}$ from all good assets, the revenue raised is at least L . This deviation generates a higher payoff if and only if

$$x'\tau\bar{p}_{con}(\tau) + (1-x')\tau\bar{v} + (1-\tau)\underline{v} > \bar{x}\bar{p}_{con}(\tau) + (1-\bar{x})(\tau\bar{v} + (1-\tau)\underline{v})$$

Using $\bar{x}\bar{p}_{con}(\tau) = L/n$, $x' = \frac{L/n - (1-\tau)\underline{v}}{\tau\bar{p}_{con}(\tau)}$, and $\bar{p}_{con}(\tau) = \underline{v} + \Delta \frac{\beta\tau}{\beta\tau+1-\tau}$, we obtain $L/n < \underline{v} \frac{1-\tau}{\beta\tau+1-\tau}$, which implies that this deviation is optimal, a contradiction. We conclude that $L/n \geq \underline{v} \frac{1-\tau}{\beta\tau+1-\tau}$ as required. Intuitively, if the shock were smaller, the seller would retain more of good assets. For the same reasons as in the benchmark, $\bar{x} = \bar{x}_{con}/n$.

Consider part (ii). We show that if $\underline{v}(1-\tau) < L/n < \underline{v}$ then the specified equilibrium indeed exists. First, note that $L/n < \underline{v} \Rightarrow \bar{x}_{div}(\tau) < 1$. Second, note that the prices in (8) are consistent with the trading strategy given by (7). Moreover, the pricing function in (8) is non-increasing. Third, we show that given the price function in (8), the seller's trading strategy in (7) is indeed optimal. Suppose $\theta = 0$. Given (8), the seller's optimal response is $v_i = \bar{v} \Rightarrow x_i = 0$ and $v_i = \underline{v} \Rightarrow x_i = \bar{x}_{div}(\tau)$, as prescribed by (7). Suppose $\theta = L$. Given (8), the seller's most profitable deviation involves selling \bar{x}_{div} from each bad asset, and the least amount of a good asset, such that she raises at least L . However, recall that by the construction of $\bar{x}_{div}(\tau)$, $(1-\tau)\underline{v} + \tau\bar{x}_{div}(\tau)\bar{p}_{div}(\tau) = L/n$. Also note that $L/n < \underline{v} \Rightarrow \bar{x}_{div}(\tau)\bar{p}_{div}(\tau) < L/n$. Therefore, the most profitable deviation generates a revenue strictly lower than L , and hence is suboptimal. This concludes part (ii).

Consider part (iii). We show that if $\underline{v} \frac{1-\tau}{\beta\tau+1-\tau} \leq L/n$ then the specified equilibrium indeed exists. The proof is as described by Proposition 1, where \bar{x}_{con} is replaced by \bar{x}_{con}/n . The only exception is that we note that as per the proof of Claim 4, the condition $\underline{v} \frac{1-\tau}{\beta\tau+1-\tau} \leq L/n$ guarantees that, if $\theta = L$, the seller has no profitable deviation. The proof that the seller has no profitable deviation when $\theta = 0$ is the same as in the proof of part (ii) above.

Finally, part (iv) follows from claims 1-4. ■

Proof of Proposition 1. From Proposition 1,

$$P_{con}(v_i, \tau) = \begin{cases} \bar{p}_{con}(\tau) & \text{if } v_i = \underline{v} \\ \beta \bar{p}_{con}(\tau) + (1 - \beta) \bar{v} & \text{if } v_i = \bar{v}. \end{cases}$$

Let $\gamma = (1 - \beta) \cdot \mathbf{1}_{\underline{v}(0)=0}$, then from Proposition 2,

$$P_{div}(\underline{v}, \tau) = \begin{cases} \gamma[\underline{v} + \Delta \frac{\tau}{\tau + \gamma(1 - \tau)}] + (1 - \gamma) \underline{v} & \text{if } L/n \leq \underline{v}(1 - \tau) \\ \{\beta \underline{v} + (1 - \beta) \bar{p}_{div}(\tau), P_{con}(\underline{v}, \tau)\} & \text{if } \underline{v}(1 - \tau) < L/n < \underline{v} \\ P_{con}(\underline{v}, \tau) & \text{if } \underline{v} \leq L/n \end{cases}$$

$$P_{div}(\bar{v}, \tau) = \begin{cases} \bar{v} + \Delta \begin{cases} \gamma[\bar{v} + \Delta \frac{\tau}{\tau + \gamma(1 - \tau)}] + (1 - \gamma) \bar{v} & \text{if } L/n \leq \bar{v}(1 - \tau) \\ \{\beta \bar{v} + (1 - \beta) \bar{p}_{div}(\tau), P_{con}(\bar{v}, \tau)\} & \text{if } \bar{v}(1 - \tau) < L/n < \bar{v} \end{cases} \\ \{\beta \bar{p}_{div}(\tau) + (1 - \beta) \bar{v}, P_{con}(\bar{v}, \tau)\} & \text{if } \bar{v}(1 - \tau) < L/n < \bar{v} \\ P_{con}(\bar{v}, \tau) & \text{if } \bar{v} \leq L/n, \end{cases}$$

and the curly brackets encompass the two possible equilibria (types-(ii) and (iii)) that can exist when $\underline{v}(1 - \tau) < L/n < \underline{v}$.

To prove (9), first suppose $\underline{v}(1 - \tau) < L/n$. It is sufficient to note that

$$\beta \underline{v} + (1 - \beta) \bar{p}_{div}(\tau) < \bar{p}_{con}(\tau)$$

and

$$\beta \bar{p}_{div}(\tau) + (1 - \beta) \bar{v} > \beta \bar{p}_{con}(\tau) + (1 - \beta) \bar{v}$$

which holds given that $\bar{p}_{div}(\tau) > \bar{p}_{con}(\tau)$. Next, suppose $L/n \leq \underline{v}(1 - \tau)$. Note that

$$\bar{p}_{con}(\tau) \geq \gamma \left[\underline{v} + \Delta \frac{\tau}{\tau + \gamma(1 - \tau)} \right] + (1 - \gamma) \underline{v} \Leftrightarrow \gamma \leq \frac{\beta \tau}{\beta \tau + (1 - \beta)(1 - \tau)}$$

$$\beta \bar{p}_{con}(\tau) + (1 - \beta) \bar{v} < \underline{v} + \Delta \frac{\tau}{\tau + \gamma(1 - \tau)} \Leftrightarrow \gamma < \frac{\beta \tau}{\beta \tau + (1 - \beta)(1 - \tau)}.$$

Therefore, this condition holds if $\underline{x}(0) > 0$ or, $\underline{x}(0) = 0$ and $1 - \beta < \frac{\beta\tau}{\beta\tau + (1-\beta)(1-\tau)} \Leftrightarrow \frac{\sqrt{1-\tau}}{\sqrt{\tau} + \sqrt{1-\tau}} < \beta$, as required. ■

Proof of Lemma 3. We first state the equilibrium selling strategies and prices, which are given by

$$x_i(v_i, \theta_i) = \begin{cases} 0 & \text{if } v_i \in \{\underline{v} + \tau\Delta, \bar{v}\} \text{ and } \theta_i = 0 \\ \bar{x} \equiv n \times \min\left\{\frac{L/n}{p_i^*(\bar{x})}, 1\right\} & \text{otherwise,} \end{cases}$$

and

$$p_i^*(x_i) = \begin{cases} \underline{v} + \Delta \frac{\tau}{\lambda_{con}^* \tau + 1 - \lambda_{con}^*} & \text{if } x_i = 0 \\ \underline{v} + \Delta \frac{\beta\tau}{\lambda_{con}^* (1-\beta)(1-\tau) + \beta} & \text{if } x_i \in (0, \bar{x}] \\ \underline{v} & \text{if } x_i > \bar{x}. \end{cases}$$

Let λ^* be the equilibrium investigation intensity. Since $c'(1) = \infty$, in any equilibrium $\lambda^* < 1$. Next, suppose $\lambda^* \in (0, 1)$. We first argue a set of results regarding the possible equilibrium selling strategies, similar to the proof of Proposition 1.

1. $x_i(\underline{v}, \theta) > 0$ and $x_i(\underline{v} + \tau\Delta, L) > 0$. To see this, note that if $\theta = L$, the seller will sell a positive amount. If $\theta_i = 0$ and $v_i = \underline{v}$, suppose that $x_i = 0$. Her payoff in this case is \underline{v} . Since $x_i(\underline{v} + \tau\Delta, L) > 0$, it must be that $p_i(x_i(\underline{v} + \tau\Delta, L)) > \underline{v}$, and so type \underline{v} has a profitable deviation to $x_i(\underline{v} + \tau\Delta, L)$, yielding a contradiction.
2. $x_i(\bar{v}, 0) \notin \{x_i(\underline{v}, L), x_i(\underline{v}, 0), x_i(\underline{v} + \tau\Delta, L)\}$. To see this, suppose on the contrary that is not the case. Then, from point 1 it must be that $x_i(\bar{v}, 0) > 0$ and $p_i(x_i(\bar{v}, 0)) < \bar{v}$. Thus, type- $(\bar{v}, 0)$ can deviate to $x'_i = 0$ and receive a payoff of \bar{v} . This yields a contradiction.
3. $x_i(\underline{v}, \theta) \in \{x_i(\underline{v} + \Delta\tau, L), x_i(\bar{v}, L)\}$. Suppose not. Then, it cannot be $x_i(\underline{v}, \theta) \notin \{x_i(\underline{v} + \Delta\tau, 0), x_i(\bar{v}, 0)\}$ since then $p(x_i(\underline{v}, \theta)) = \underline{v}$ and the seller's payoff is \underline{v} . Indeed, since $x_i(\underline{v} + \tau\Delta, L) > 0$ and $p_i(x_i(\underline{v} + \tau\Delta, L)) > \underline{v}$, the seller has a profitable deviation to $x_i(\underline{v} + \tau\Delta, L) > 0$. However, if $x_i(\underline{v}, \theta) \in \{x_i(\underline{v} + \Delta\tau, 0), x_i(\bar{v}, 0)\}$, then based on point 2, it must be $x_i(\underline{v}, \theta) = x_i(\underline{v} + \Delta\tau, 0) \neq x_i(\bar{v}, 0)$. Note that if $p_i(x_i(\underline{v} + \Delta\tau, 0)) < \underline{v} + \Delta\tau$, we get a contradiction since the type $(\underline{v} + \Delta\tau, 0)$ would have a profitable deviation to $x = 0$. However, since $x_i(\underline{v}, \theta) = x_i(\underline{v} + \Delta\tau, 0) \neq x_i(\bar{v}, 0)$, $p_i(x_i(\underline{v} + \Delta\tau, 0)) \geq \underline{v} + \Delta\tau$ requires

$x_i(\underline{v} + \Delta\tau, 0) = x_i(\bar{v}, L)$. But this contradicts $x_i(\underline{v}, \theta) \notin \{x_i(\underline{v} + \Delta\tau, L), x_i(\bar{v}, L)\}$. Therefore, $x_i(\underline{v}, \theta) \in \{x_i(\underline{v} + \Delta\tau, L), x_i(\bar{v}, L)\}$ as required.

4. $x_i(\underline{v}, 0) = x_i(\underline{v}, L) = \bar{x} > 0$. Suppose on the contrary $x_i(\underline{v}, 0) \neq x_i(\underline{v}, L)$. Given point 3, it must be $x_i(\underline{v} + \Delta\tau, L) \neq x_i(\bar{v}, L)$. Moreover, in following both strategies, the seller either obtains the highest revenue, or she obtains a revenue of at least L . Therefore, the liquidity constraint does not bind, and the payoff of the strategies must generate the same payoff. That is, let $x_\theta = x_i(\underline{v}, \theta)$ then

$$\begin{aligned} x_0 p_i(x_0) + (1 - x_0)\underline{v} &= x_L p_i(x_L) + (1 - x_L)\underline{v} \Leftrightarrow \\ x_0 p_i(x_0) - x_L p_i(x_L) &= (x_0 - x_L)\underline{v}. \end{aligned}$$

There are two cases. First, if $x_0 = x_i(\underline{v} + \Delta\tau, L)$ (and therefore, $x_L = x_i(\bar{v}, L)$) then it must be

$$\begin{aligned} x_0 p_i(x_0) - x_L p_i(x_L) &\geq (x_0 - x_L)(\underline{v} + \Delta\tau) \text{ and,} \\ x_0 p_i(x_0) - x_L p_i(x_L) &\leq (x_0 - x_L)\bar{v} \end{aligned}$$

which holds if and only if

$$(x_0 - x_L)\bar{v} \geq (x_0 - x_L)\underline{v} \geq (x_0 - x_L)(\underline{v} + \Delta\tau),$$

which can never holds. Second, if $x_0 = x_i(\bar{v}, L)$ (and therefore, $x_L = x_i(\underline{v} + \Delta\tau, L)$) then it must be

$$\begin{aligned} x_0 p_i(x_0) - x_L p_i(x_L) &\leq (x_0 - x_L)(\underline{v} + \Delta\tau) \text{ and,} \\ x_0 p_i(x_0) - x_L p_i(x_L) &\geq (x_0 - x_L)\bar{v} \end{aligned}$$

which holds if and only if

$$(x_0 - x_L)(\underline{v} + \Delta\tau) \geq (x_0 - x_L)\underline{v} \geq (x_0 - x_L)\bar{v},$$

which also can never hold. Therefore, a contradiction.

5. $x_i(\underline{v} + \Delta\tau, L) = \bar{x}$. Suppose instead $x_i(\underline{v} + \Delta\tau, L) = x'_i \neq \bar{x}$. Since $x'_i \neq \bar{x}$, based on point 4 we have $p_i(x'_i) \geq \underline{v} + \Delta\tau$. Moreover, note that either $x'_i p_i(x'_i) \geq L/n$ or x'_i generates the highest revenue that can be obtained in equilibrium. There are two cases:

- (a) Suppose $p_i(\bar{x}) < p_i(x'_i)$. If $x'_i \geq \bar{x}$ then type- $(\underline{v}, 0)$ has a profitable deviation to x'_i , since she can sell more shares for a higher price. If $x'_i < \bar{x}$ then if $x_i(\bar{v}, L) = \bar{x}$ then type- (\bar{v}, L) has a profitable deviation to x'_i (which satisfies her liquidity needs). If instead $x_i(\bar{v}, L) \neq \bar{x}$ then $p_i(\bar{x}) = \underline{v}$, and type- $(\underline{v}, 0)$ has a profitable deviation to x'_i , a contradiction.
- (b) Suppose $p_i(\bar{x}) \geq p_i(x'_i)$. Then, type- (\bar{v}, L) must play \bar{x} with positive probability. By revealed preference, this means that

$$\bar{x}p_i(\bar{x}) - x'_i p_i(x'_i) \geq (\bar{x} - x'_i) \bar{v}.$$

Since type \underline{v} also weakly prefers \bar{x} over x'_i ,

$$\bar{x}p_i(\bar{x}) - x'_i p_i(x'_i) \geq (\bar{x} - x'_i) \underline{v}.$$

However, type $\underline{v} + \Delta\tau$ weakly prefer x'_i over \bar{x} ,

$$\bar{x}p_i(\bar{x}) - x'_i p_i(x'_i) \leq (\bar{x} - x'_i) (\underline{v} + \Delta\tau).$$

The combination of the three conditions implies $\bar{x} - x'_i = 0$, a contradiction.

6. $p_i(\bar{x}) < \underline{v} + \tau\Delta$. Based on points 1-5 and the application of Bayes' rule,

$$p_i(\bar{x}) \leq \max_{\gamma \in [0,1]} \left\{ \underline{v} + \Delta \frac{(1-\beta)(1-\lambda^*)\gamma\tau + \beta\tau}{(1-\beta)[\lambda^*(1-\tau) + (1-\lambda^*)\gamma] + \beta} \right\}.$$

Indeed, at best, type- (\bar{v}, L) chooses \bar{x} w.p. one, and type- $(\underline{v} + \Delta\tau, 0)$ chooses \bar{x} w.p. γ . Note that since $\tau \in (0, 1)$ and $\lambda^* \in (0, 1)$, for every $\gamma \in [0, 1]$ the RHS is strictly smaller than $\underline{v} + \Delta\tau$.

7. $x_i(\bar{v}, L) = \bar{x}$. Suppose instead $x_i(\bar{v}, L) = x'_i \neq \bar{x}$. Based on points 4 and 6, it must be that $p_i(x'_i) > \underline{v} + \Delta\tau > p_i(\bar{x})$. Therefore, type- $(\underline{v} + \Delta\tau, L)$ has a profitable deviation to

x'_i since it leads to a trading profit and also satisfies her liquidity needs, a contradiction.

8. $x_i(\underline{v} + \Delta\tau, 0) \neq \bar{x}$. Suppose instead that $x_i(\underline{v} + \Delta\tau, 0) = \bar{x}$. Based on point 6, $p_i(\bar{x}) < \underline{v} + \Delta\tau$. Therefore, type- $(\underline{v} + \Delta\tau, 0)$ has strict incentives to choose $x_i = 0$ instead of \bar{x} , a contradiction.

9. $x_i(\bar{v}, 0) = 0$. Suppose instead $x_i(\bar{v}, 0) = x'_i > 0$. Then, $p_i(x'_i) = \bar{v}$. Therefore, it must be $x'_i \neq x_i(\underline{v} + \Delta\tau, 0)$. Based on points 1-8, either $x_i(\underline{v} + \Delta\tau, 0) = 0$ or $p_i(x_i(\underline{v} + \Delta\tau, 0)) = \underline{v} + \Delta\tau$. This implies that the equilibrium payoff of type- $(\underline{v} + \Delta\tau, 0)$ is $\underline{v} + \Delta\tau$. However, in this case, type- $(\underline{v} + \Delta\tau, 0)$ has strict incentives to play x'_i and receive a payoff strictly higher than $\underline{v} + \Delta\tau$, a contradiction.

These points show that in any equilibrium with $\lambda^* \in (0, 1)$, we have $x_i(\bar{v}, L) = x_i(\underline{v} + \Delta\tau, L) = x_i(\underline{v}, L) = x_i(\underline{v}, 0) = \bar{x} > 0$, $x_i(\bar{v}, 0) = 0$, and $x_i(\underline{v} + \Delta\tau, 0) \neq \bar{x}$. Now, given this, without loss of generality (for investigation incentives) we consider the case where $x_i(\underline{v} + \Delta\tau, 0) = 0$. Note that the selling strategies in the proposition satisfy the conditions of the previous part of the proof. Given these selling strategies, the prices satisfy Bayes' rule. Showing $\bar{x} = n \times \min \left\{ \frac{L/n}{p_i^*(\bar{x})}, 1 \right\}$ can be done using the same arguments made in the proof of Lemma 1.

The seller's expected payoff from choosing $\lambda \in [0, 1]$ is

$$\begin{aligned} \Pi(\lambda) = & \lambda \left[\begin{array}{l} \tau[(1 - \beta)n\bar{v} + \beta(\bar{x}p_i^*(\bar{x}) + (n - \bar{x})\bar{v})] \\ + (1 - \tau)[\bar{x}p_i^*(\bar{x}) + (n - \bar{x})\underline{v}] \end{array} \right] \\ & + (1 - \lambda) \left[\begin{array}{l} (1 - \beta)n(\underline{v} + \Delta\tau) \\ + \beta[\bar{x}p_i^*(\bar{x}) + (n - \bar{x})(\underline{v} + \Delta\tau)] \end{array} \right] - c(\lambda) \end{aligned}$$

The first order condition implies

$$\begin{aligned} \left[\begin{array}{l} \tau[(1 - \beta)n\bar{v} + \beta(\bar{x}p_i^*(\bar{x}) + (n - \bar{x})\bar{v})] \\ + (1 - \tau)[\bar{x}p_i^*(\bar{x}) + (n - \bar{x})\underline{v}] \end{array} \right] - \left[\begin{array}{l} (1 - \beta)n(\underline{v} + \Delta\tau) \\ + \beta[\bar{x}p_i^*(\bar{x}) + (n - \bar{x})(\underline{v} + \Delta\tau)] \end{array} \right] = c'(\lambda) \Leftrightarrow \\ (1 - \tau)(1 - \beta)(p_i^*(\bar{x}) - \underline{v})\bar{x} = c'(\lambda) \Leftrightarrow \\ (1 - \tau)(1 - \beta)\beta\tau\Delta n \min \left\{ \begin{array}{l} \frac{L/n}{\underline{v}[\lambda^*(1 - \beta)(1 - \tau) + \beta] + \Delta\beta\tau}, \\ \frac{1}{\lambda^*(1 - \beta)(1 - \tau) + \beta} \end{array} \right\} = c'(\lambda) \end{aligned}$$

Therefore, in equilibrium, if $\lambda^* \in (0, 1)$ then it must satisfy (11). Note that the RHS of (11) is decreasing in λ^* and the LHS is increasing in λ^* (since $c'' > 0$ by assumption), and so if a

solution exists, it is unique. Furthermore, if $\beta < 1$ then at $\lambda^* = 0$, the RHS is greater and as $\lambda^* = 1$, the LHS is greater. Thus, if $\beta < 1$ then there exists a unique λ^* satisfying (11).

Suppose $\beta = 1$. The analysis above shows that we cannot have $\lambda^* > 0$ in equilibrium. Indeed, if $\lambda^* > 0$ then λ^* must satisfy (11). However, $\beta = 1$ implies that if λ^* satisfies (11) then $\lambda^* = 0$. Therefore, if $\beta = 1$ it must be $\lambda^* = 0$. Consider the prices and quantities in the proposition when $\lambda_{con}^* = 0$. Clearly, given $\lambda = 0$, the selling strategy is weakly optimal and the prices are consistent with these strategies. It is left to verify the seller has no incentives to deviate to $\lambda > 0$. Indeed, substituting $\beta = 1$ and $p_i^*(\bar{x}) = \underline{v} + \tau\Delta$ into $\Pi(\lambda)$ above yields $\Pi(\lambda) = n(\tau\Delta + \underline{v}) - c(\lambda)$, which implies that no $\lambda > 0$ is optimal, as required. ■

Proof of Lemma 4. We first note that the convexity of $c(\cdot)$ implies that, in any equilibrium, all assets are investigated with equal intensity. Indeed, suppose on the contrary in equilibrium a measure $ng(\lambda)$ of assets are investigated with intensity λ . Then, by the law of large numbers, the aggregate measure of investigated assets is $\bar{\lambda} \equiv n \int \lambda g(\lambda) d\lambda$, and the aggregate cost borne by the seller is $n \int c(\lambda) g(\lambda) d\lambda$. Consider a deviation to investigating each asset w.p. $\bar{\lambda}/n$. The aggregated measure of investigated assets is $\bar{\lambda}$, and the aggregate cost borne by the seller is $nc(\bar{\lambda}/n)$. Note that by Jensen's inequality, $c(\bar{\lambda}/n) = c(\int \lambda g(\lambda) d\lambda) < \int c(\lambda) g(\lambda) d\lambda$, which implies that this deviation is profitable.

Next, we prove part (i). Suppose in equilibrium λ^* is such that $L/n \leq \lambda^*(1 - \tau)\underline{v}$, then the seller can satisfy the shock from selling only bad assets. Let

$$x_i(v_i, \theta) = \begin{cases} 0 & \text{if } v_i \in \{\underline{v} + \tau\Delta, \bar{v}\} \\ 1 & \text{if } v_i = \underline{v}, \end{cases}$$

and

$$p_i^*(x_i) = \begin{cases} \underline{v} + \Delta \frac{\tau}{\lambda^*\tau + 1 - \lambda^*} & \text{if } x_i = 0 \\ \underline{v} & \text{if } x_i > 0. \end{cases}$$

Then, the proposed selling strategies are consistent with such an equilibrium given prices, and prices are consistent with the proposed strategies by Bayes' rule.

Suppose the seller chooses λ , where we allow for $\lambda \neq \lambda^*$. Let $\Pi(\lambda, \lambda^*)$ denote her payoff from this deviation. There are two cases. First, if $L/n \leq \lambda(1 - \tau)\underline{v}$ then it is optimal for the seller to choose the same selling strategies as under the equilibrium λ^* , i.e. sells bad assets and

retains all others. In this case

$$\Pi(\lambda, \lambda^*)/n = \underline{v} + \Delta\tau - c(\lambda). \quad (18)$$

Second, suppose $L/n > \lambda(1 - \tau)\underline{v}$. If $\theta = 0$ then the seller strictly prefers to retain all assets not identified as bad, and is indifferent between selling and retaining bad assets. Let an “unidentified asset” be an asset of unknown quality, i.e. for which investigation has been unsuccessful. If $\theta = L$ then the highest payoff (subject to meeting the liquidity need) is generated by a strategy in which the seller sells all bad assets, sells $\min\{\frac{L/n - \lambda(1 - \tau)\underline{v}}{(1 - \lambda)\underline{v}}, 1\}$ from all unidentified assets, and sells $\min\{\max\{\frac{L/n - (1 - \lambda\tau)\underline{v}}{\lambda\tau\underline{v}}, 0\}, 1\}$ from all good assets. Since $L/n < (1 - \tau)\underline{v} \Rightarrow L/n < (1 - \lambda\tau)\underline{v}$, the former term is $\frac{L/n - \lambda(1 - \tau)\underline{v}}{(1 - \lambda)\underline{v}} < 1$ and the last term is zero (i.e. the seller never has to sell good assets). In this case,

$$\begin{aligned} \Pi(\lambda, \lambda^*)/n &= -c(\lambda) + (1 - \beta)(\underline{v} + \Delta\tau) \\ &+ \beta \left[\begin{aligned} &\lambda\tau(\underline{v} + \Delta) + \lambda(1 - \tau)\underline{v} \\ &+ (1 - \lambda) \left(\frac{L/n - \lambda(1 - \tau)\underline{v}}{(1 - \lambda)\underline{v}}\underline{v} + \left(1 - \frac{L/n - \lambda(1 - \tau)\underline{v}}{(1 - \lambda)\underline{v}}\right)(\underline{v} + \Delta\tau) \right) \end{aligned} \right] \end{aligned}$$

Combining the two cases we have,

$$\frac{\partial \Pi(\lambda, \lambda^*)}{\partial \lambda} = n \times \begin{cases} -c'(\lambda) + \beta\tau\Delta(1 - \tau) & \text{if } \lambda < \frac{L/n}{(1 - \tau)\underline{v}} \\ -c'(\lambda) & \text{if } \lambda \geq \frac{L/n}{(1 - \tau)\underline{v}}. \end{cases}$$

Since

$$-c' \left(\frac{L/n}{(1 - \tau)\underline{v}} \right) + \beta\tau\Delta(1 - \tau) \leq 0 \Leftrightarrow \Lambda(\beta)(1 - \tau)\underline{v} \leq L/n,$$

(note that $c'(1) = \infty \Rightarrow \Lambda(\beta) < 1$) the first order condition implies

$$\lambda^* = \begin{cases} \Lambda(\beta) & \text{if } \Lambda(\beta)(1 - \tau)\underline{v} < L/n \\ \frac{L/n}{(1 - \tau)\underline{v}} & \text{if } \Lambda(\beta)(1 - \tau)\underline{v} \geq L/n \end{cases}$$

Therefore, if $\Lambda(\beta)(1 - \tau)\underline{v} \geq L/n$ then there is an equilibrium with $\lambda_{div}^* = \frac{L/n}{(1 - \tau)\underline{v}}$, as required for part (i)

Next, consider part (ii). Suppose on the contrary $L/n < (1 - \tau)\underline{v}$ and there is an equilibrium

with $\lambda^* = 0$. Since $\beta > 0$ and $L/n < (1 - \tau)\underline{v}$, and since the equilibrium is symmetric, there is $\bar{x} > 0$ on the equilibrium path such that $\bar{x}p(\bar{x}) \geq L/n$. Since $\lambda^* = 0$ and the market has rational expectations, it must be $p(\bar{x}) = \underline{v} + \tau\Delta$. Note that $p(1) \geq \underline{v}$. Suppose the seller chooses $\lambda > 0$ and the following selling strategy: (i) if the seller does not need liquidity, she sells \bar{x} units from all assets; (ii) if the seller needs liquidity, she sells \bar{x} units from all unidentified assets and none from identified good assets. Moreover, the seller sells \bar{x} units from a fraction $\eta \in (0, 1)$ of identified bad assets, and fully sells a fraction $1 - \eta$ of identified bad assets. Using $\bar{x}p(\bar{x}) \geq L/n$ and $p(1) \geq \underline{v}$, the revenue generated by this strategy is

$$\begin{aligned} & n\lambda(1 - \tau)[\eta\bar{x}p(\bar{x}) + (1 - \eta)p(1)] + n(1 - \lambda)\bar{x}p(\bar{x}) \\ \geq & n\lambda(1 - \tau)(1 - \eta)\underline{v} + [(1 - \lambda) + \lambda(1 - \tau)\eta]L \end{aligned}$$

Note that

$$\lambda(1 - \tau)(1 - \eta)\underline{v} + [(1 - \lambda) + \lambda(1 - \tau)\eta]L/n \geq L/n \Leftrightarrow \eta \leq \frac{\underline{v} - \frac{L/n}{1 - \tau}}{\underline{v} - L/n}$$

where $L/n < (1 - \tau)\underline{v} \Rightarrow \frac{\underline{v} - \frac{L/n}{1 - \tau}}{\underline{v} - L/n} \in (0, 1)$. Therefore, if $\eta = \frac{\underline{v} - \frac{L/n}{1 - \tau}}{\underline{v} - L/n}$ then using this strategy the seller can generate enough revenue to meet her liquidity needs. The seller's profit is:

$$\begin{aligned} \Pi(\lambda, 0)/n &= \beta \left[\begin{aligned} & (1 - \lambda)(\underline{v} + \tau\Delta) + \lambda\tau(\underline{v} + \Delta) \\ & + \lambda(1 - \tau)[\eta(\bar{x}p(\bar{x}) + (1 - \bar{x})\underline{v}) + (1 - \eta)p(1)] \end{aligned} \right] \\ &+ (1 - \beta)(\underline{v} + \tau\Delta) - c(\lambda). \end{aligned}$$

Note that

$$\frac{\partial \Pi(\lambda, 0)}{\partial \lambda} = (1 - \tau)\beta n[\eta\bar{x}\tau\Delta + (1 - \eta)(p(1) - \underline{v})] - nc'(\lambda).$$

Therefore, $\frac{\partial \Pi(\lambda, 0)}{\partial \lambda}|_{\lambda=0} > 0$, which implies that the seller has a profitable deviation to $\lambda > 0$, a contradiction. This proves part (ii).

Finally, consider part (iii). We argue that there exists an equilibrium with $\lambda^* = 0$. Let the seller strategy in this equilibrium be selling $\bar{x} = \frac{L/n}{\underline{v} + \tau\Delta} < 1$ unit from each asset, and prices are

monotonic and given by

$$p_i^*(x_i) = \begin{cases} \underline{v} + \tau\Delta & \text{if } x_i \leq \bar{x} \\ \underline{v} & \text{if } x_i > \bar{x}. \end{cases}$$

Suppose the seller deviates and chooses $\lambda > 0$. Note that in order to raise enough liquidity, the seller can follow the equilibrium strategy and sell \bar{x} from each asset. This strategy generates an average profit of $n(\underline{v} + \tau\Delta)$. If so, the seller is better off choosing $\lambda = 0$. Thus any other strategy that generates an expected payoff strictly higher than $n(\underline{v} + \tau\Delta)$ (and at the same time at least L of revenue) requires selling less than \bar{x} from identified good assets and more than \bar{x} from identified bad assets or unidentified assets. Since $p_i^*(x_i) < \underline{v} + \Delta$, the strategy that maximizes the profit from this deviation involves fully selling all identified bad assets, which generates a revenue of $n\lambda(1 - \tau)\underline{v}$. Consider a strategy that involves selling $\bar{x} + \varepsilon$ from each unidentified asset and $\bar{x} - \delta$ from each identified good asset, where $\varepsilon, \delta > 0$. The expected payoff of the seller from this deviation is

$$\begin{aligned} \Pi(\lambda, 0) &= n\lambda(1 - \tau)\underline{v} + n\lambda\tau[(\bar{x} - \delta)p_i^*(\bar{x} - \delta) + (1 - (\bar{x} - \delta))\bar{v}] \\ &\quad + n(1 - \lambda)[(\bar{x} + \varepsilon)p_i^*(\bar{x} + \varepsilon) + (1 - (\bar{x} + \varepsilon))(\underline{v} + \tau\Delta)] - nc(\lambda) \end{aligned}$$

However, $\Pi(\lambda, 0) > n(\underline{v} + \tau\Delta)$ if and only if

$$\begin{aligned} \left(\begin{array}{l} \lambda(1 - \tau)\underline{v} + \lambda\tau[(\bar{x} - \delta)(\underline{v} + \tau\Delta) + (1 - (\bar{x} - \delta))(\underline{v} + \Delta)] \\ + (1 - \lambda)[(\bar{x} + \varepsilon)\underline{v} + (1 - (\bar{x} + \varepsilon))(\underline{v} + \tau\Delta)] - c(\lambda) \end{array} \right) &> \underline{v} + \tau\Delta \Leftrightarrow \\ \lambda[(\bar{x} - \delta)\tau + (1 - (\bar{x} - \delta))] + (1 - \lambda)(1 - (\bar{x} + \varepsilon)) - 1 &> \frac{c(\lambda)}{\tau\Delta} \end{aligned}$$

which never holds. Therefore, a profitable deviation does not exist. ■

Proof of Proposition 2. Based on Proposition 3, in any equilibrium under concentration, $\lambda_{con}^*(\beta)$ is unique and given by the solution of (11). Therefore, $\lim_{\beta \rightarrow 1} \lambda_{con}^*(\beta) = 0$. Based on parts (i) and (ii) of Proposition 4, if $L/n < \Lambda(\beta) \cdot (1 - \tau)\underline{v}$ then an equilibrium always exists under diversification, and in any equilibrium and for any $\beta \in (0, 1]$ we have $\lambda_{div}^*(\beta) > 0$. Since $\Lambda(\beta)$ is increasing in β , there exists $\bar{\beta} < 1$ such that if $L/n \leq \Lambda(\bar{\beta}) \cdot (1 - \tau)\underline{v}$ and $\beta > \bar{\beta}$ then $\lambda_{div}^*(\beta) > \lambda_{con}^*(\beta)$, as required. ■

Proof of Proposition 3. The proof of Proposition 3 implicitly assumes that $x_i(\underline{v} + \Delta\tau, 0) = \{0\}$.²⁴ To maximize price informativeness under concentration (which goes against our results that price informativeness is lower under concentration), we suppose $x_i(\underline{v} + \Delta\tau, 0) = \{\varepsilon\}$, where $0 \leq \varepsilon < \bar{x}$ is arbitrarily small such that no deviation for other types is profitable and

$$x_i(v_i, \theta_i) = \begin{cases} 0 & \text{if } v_i = \underline{v} \text{ and } \theta_i = 0 \\ \varepsilon & \text{if } v_i = \underline{v} + \tau\Delta \text{ and } \theta_i = 0 \\ \bar{x}_{con} \equiv n \times \min\left\{\frac{L/n}{\bar{p}_{con}}, 1\right\} & \text{otherwise,} \end{cases}$$

$$p_{con,info}^*(x_i) = \begin{cases} \bar{v} & \text{if } x_i = 0 \\ \underline{v} + \tau\Delta & \text{if } x_i \in (0, \varepsilon] \\ \bar{p}_{con} = \underline{v} + \Delta \frac{\beta\tau}{\lambda_{con}^*(1-\beta)(1-\tau)+\beta} & \text{if } x_i \in (\varepsilon, \bar{x}_{con}] \\ \underline{v} & \text{if } x_i > \bar{x}_{con}. \end{cases}$$

Under this formulation,

$$P_{con,info}(v_i, \tau) = \begin{cases} \beta\bar{p}_{con} + (1-\beta)[\lambda_{con}^*\bar{p}_{con} + (1-\lambda_{con}^*)(\underline{v} + \tau\Delta)] & \text{if } v_i = \underline{v} \\ \beta\bar{p}_{con} + (1-\beta)[\lambda_{con}^*\bar{v} + (1-\lambda_{con}^*)(\underline{v} + \tau\Delta)] & \text{if } v_i = \bar{v}. \end{cases}$$

Next, consider diversification when $L/n \leq \Lambda(\beta)(1-\tau)\underline{v}$. Note that according to Proposition 4, if $L/n \leq \Lambda(\beta)(1-\tau)\underline{v}$, there always exists an equilibrium in which only identified bad firms are sold. Consider equilibria under diversification in which the seller must sell some unidentified assets to satisfy her shock, i.e., $\lambda^*(1-\tau)\underline{v} < L/n$. Note that

$$L/n \leq \Lambda(\beta)(1-\tau)\underline{v} < (1-\tau)\underline{v} = \min_{\lambda} \{(1-\tau\lambda)\underline{v}\}.$$

Therefore, if $L/n \leq \Lambda(\beta) \cdot (1-\tau)\underline{v}$, good assets are never sold. Then the seller holds only two types of assets that she may potentially sell. Let \hat{n} denote the mass of assets not identified as a good, a fraction $\hat{\tau}$ of which are valued at \hat{v} and a fraction $1 - \hat{\tau}$ of which are valued at \underline{v} ,

²⁴This assumption has no effect on the analysis of λ_{con}^* , because regardless of whether she retains uninvestigated firms, she obtains their expected value, $\underline{v} + \Delta\tau$.

where

$$\begin{aligned}\hat{\tau} &= \frac{1 - \lambda^*}{\lambda^*(1 - \tau) + 1 - \lambda^*} = \frac{1 - \lambda^*}{1 - \tau\lambda^*} \\ \widehat{\bar{v}} &= \underline{v} + \widehat{\Delta} \\ \widehat{\Delta} &= \tau\Delta \\ \hat{n} &= n(1 - \tau\lambda^*).\end{aligned}$$

Note that

$$\lambda^*(1 - \tau)\underline{v} < L/n \leq (1 - \tau\lambda^*)\underline{v} \Leftrightarrow (1 - \hat{\tau})\underline{v} < L/\hat{n} \leq \underline{v}.$$

We apply parts (ii) and (iii) of Proposition 2, where τ is replaced by $\hat{\tau}$, τ^* by $\hat{\tau}^*$, \bar{v} by $\widehat{\bar{v}}$, Δ by $\widehat{\Delta}$, and n by \hat{n} . The only exception is $p_i(0)$, which is different but has no effect on the decision to investigate. That is, there are only two possible equilibria of this type. Let $\bar{x}^* = \bar{x}_{div}(\hat{\tau}^*)$ and $\bar{p}^* = \bar{p}_{div}(\hat{\tau}^*)$ if the equilibrium is type-(ii) and let $\bar{x}^* = \bar{x}_{con}(\hat{\tau}^*)/n$ and $\bar{p}^* = \bar{p}_{con}(\hat{\tau}^*)$ if the equilibrium is type-(iii), as described in Proposition 2.

Overall, if $L/n \leq \Lambda(\beta) \cdot (1 - \tau)\underline{v}$, there are three cases to consider, one for each type of equilibrium that can emerge under diversification. We show that in each of these equilibria, price informativeness is higher under diversification when $\beta = 1$. If true, part (i) is proved. Note that if $\beta = 1$ then

$$P_{con,info}(\underline{v}, \tau) = P_{con,info}(\bar{v}, \tau) = \bar{p}_{con} = \underline{v} + \Delta\tau,$$

and according to part (ii) of Proposition 4, $\lambda_{div}^* > 0$ in any equilibrium. There are three cases to consider. For simplicity, we will refer to an equilibrium with the properties of parts (i), (ii), and (iii) of Lemma 2 as type-(i), (ii), and (iii) equilibria.

- i. Consider a type-(i) equilibrium, if it exists. In the proof of Proposition 4 there is an implicit assumption that bad assets are always fully sold, even if the seller does not suffer a shock and is indifferent between selling and not selling.²⁵ To minimize price informativeness under diversification (which goes against our results that price informativeness is higher under diversification), we suppose that the seller fully retains bad assets if

²⁵This assumption has no effect on the analysis of λ_{div}^* , because regardless of whether she fully sells bad firms, she obtains their fundamental value, \underline{v} .

there is no shock. Under this assumption, $x_i(\underline{v}, 0) = 0$, and similar to Proposition 4, if $L/n \leq \Lambda(\beta) \cdot (1 - \tau)\underline{v}$ then prices satisfy

$$P_{div,info}^*(x_i) = \begin{cases} \underline{v} + \Delta \frac{\tau}{\lambda_{div}^*(\tau + (1-\tau)(1-\beta)) + 1 - \lambda_{div}^*} & \text{if } x_i = 0 \\ \underline{v} & \text{if } x_i > 0. \end{cases}$$

Therefore,

$$P_{div,info}(v_i, \tau) = \begin{cases} \lambda_{div}^* (\beta \underline{v} + (1 - \beta) P_{div,info}^*(0)) + (1 - \lambda_{div}^*) P_{div,info}^*(0) & \text{if } v_i = \underline{v} \\ P_{div,info}^*(0) & \text{if } v_i = \bar{v}. \end{cases}$$

If $\beta = 1$ then

$$\begin{aligned} P_{div,info}(\underline{v}, \tau) &< \underline{v} + \Delta \tau < P_{div,info}(\bar{v}, \tau) \Leftrightarrow \\ \lambda_{div}^* \underline{v} + (1 - \lambda_{div}^*) P_{div,info}^*(0) &< \underline{v} + \Delta \tau < P_{div,info}^*(0) \Leftrightarrow \\ \underline{v} + \Delta \tau &< P_{div,info}^*(0) < \underline{v} + \frac{\Delta \tau}{1 - \lambda_{div}^*} \Leftrightarrow \\ 1 &< \frac{1}{\lambda_{div}^* \tau + 1 - \lambda_{div}^*} < \frac{1}{1 - \lambda_{div}^*} \end{aligned}$$

which always holds. Note that this equilibrium always exists under the condition of the proposition, and here the inequalities hold strictly.

- ii. Consider a type-(ii) equilibrium, if it exists, and let λ_{ii}^* be the corresponding λ_{div}^* . Recall that in this equilibrium, if $\beta = 1$ then

$$x_{div}^*(v_i, \theta) = \begin{cases} 0 & \text{if } v_i = \bar{v}, \text{ or } v_i = \underline{v} + \tau \Delta \text{ and } \theta = 0 \\ \frac{\underline{v} + \frac{L/n - \underline{v}}{\tau}}{\bar{p}_{div}} = & \text{if } v_i = \underline{v} \text{ and } \theta = 0, \text{ or } v_i = \underline{v} + \tau \Delta \text{ and } \theta = L \\ \frac{L/n - \underline{v}(1-\tau)\lambda_{ii}^*}{1 - \lambda_{ii}^*} \frac{1}{\bar{p}_{div}} < 1 & \\ 1 & \text{if } v_i = \underline{v} \text{ and } \theta = L, \end{cases} \quad (19)$$

and prices of asset i are:

$$p_{div,info}^*(x_i) = \begin{cases} \underline{v} + \Delta \frac{\lambda_{ii}^* \tau + (1-\beta)(1-\lambda_{ii}^*) \tau}{\lambda_{ii}^* \tau + (1-\beta)(1-\lambda_{ii}^*)} & \text{if } x_i = 0 \\ \bar{p}_{div} = \underline{v} + \Delta \frac{\beta(1-\lambda_{ii}^*) \tau}{\beta(1-\lambda_{ii}^*) + (1-\beta)\lambda_{ii}^*(1-\tau)} & \text{if } x_i \in (0, \bar{x}_{div}], \\ \underline{v} & \text{if } x_i > \bar{x}_{div}. \end{cases} \quad (20)$$

Therefore,

$$P_{div,info}(v_i, \tau) = \begin{cases} \beta [(1 - \lambda_{ii}^*) \bar{p}_{div} + \lambda_{ii}^* \underline{v}] + (1 - \beta) [(1 - \lambda_{ii}^*) p_{div,info}^*(0) + \lambda_{ii}^* \bar{p}_{div}] & \text{if } v_i = \underline{v} \\ \beta [(1 - \lambda_{ii}^*) \bar{p}_{div} + \lambda_{ii}^* p_{div,info}^*(0)] + (1 - \beta) p_{div,info}^*(0) & \text{if } v_i = \bar{v}. \end{cases}$$

If $\beta = 1$ then

$$\begin{aligned} P_{div,info}(\underline{v}, \tau) &< \underline{v} + \Delta \tau < P_{div,info}(\bar{v}, \tau) \Leftrightarrow \\ (1 - \lambda_{ii}^*) \bar{p}_{div} + \lambda_{ii}^* \underline{v} &< \underline{v} + \Delta \tau < (1 - \lambda_{ii}^*) \bar{p}_{div} + \lambda_{ii}^* p_{div,info}^*(0) \Leftrightarrow \\ (1 - \lambda_{ii}^*) (\underline{v} + \Delta \tau) + \lambda_{ii}^* \underline{v} &< \underline{v} + \Delta \tau < (1 - \lambda_{ii}^*) (\underline{v} + \Delta \tau) + \lambda_{ii}^* (\underline{v} + \Delta), \end{aligned}$$

which always holds given that $\lambda_{ii}^* > 0$.

- iii. Consider a type-(iii) equilibrium, if it exists, and let λ_{iii}^* be the corresponding λ_{div}^* . Focusing on the selling strategy that minimizes price informativeness, this equilibrium involves selling strategies

$$x^*(v_i, \theta_i) = \begin{cases} 0 & \text{if } v_i = \bar{v}, \text{ or } v_i = \underline{v} + \tau \Delta \text{ and } \theta = 0 \\ \bar{x}_{con} \equiv n \times \min \left\{ \frac{L/n}{\bar{p}_{con}}, 1 \right\} & \text{otherwise,} \end{cases} \quad (21)$$

and prices

$$p_{div,info}^*(x_i) = \begin{cases} \underline{v} + \Delta \frac{\lambda_{iii}^* \tau + (1-\beta)(1-\lambda_{iii}^*) \tau}{\lambda_{iii}^* \tau + (1-\beta)(1-\lambda_{iii}^*)} & \text{if } x_i = 0 \\ \bar{p}_{div} = \underline{v} + \Delta \frac{\beta \tau}{\lambda_{iii}^* (1-\beta)(1-\tau) + \beta} & \text{if } x_i \in (0, \bar{x}_{con}], \\ \underline{v} & \text{if } x_i > \bar{x}_{con}(\tau). \end{cases} \quad (22)$$

Therefore,

$$P_{div,info}(v_i, \tau) = \begin{cases} \beta \bar{p}_{div} + (1 - \beta) [\lambda_{iii}^* \bar{p}_{div} + (1 - \lambda_{iii}^*) p_{div,info}^*(0)] & \text{if } v_i = \underline{v} \\ \beta [(1 - \lambda_{iii}^*) \bar{p}_{div} + \lambda_{iii}^* p_{div,info}^*(0)] + (1 - \beta) p_{div,info}^*(0) & \text{if } v_i = \bar{v}. \end{cases}$$

If $\beta = 1$ then

$$\begin{aligned} P_{div,info}(\underline{v}, \tau) &\leq \underline{v} + \Delta\tau < P_{div,info}(\bar{v}, \tau) \Leftrightarrow \\ \bar{p}_{div} &\leq \underline{v} + \Delta\tau < (1 - \lambda_{iii}^*) \bar{p}_{div} + \lambda_{iii}^* p_{div,info}^*(0) \Leftrightarrow \\ \underline{v} + \Delta\tau &\leq \underline{v} + \Delta\tau < (1 - \lambda_{iii}^*) (\underline{v} + \Delta\tau) + \lambda_{iii}^* (\underline{v} + \Delta), \end{aligned}$$

which always holds given that $\lambda_{iii}^* > 0$.

Consider part (ii). Consider the type-(i) equilibrium described in the proof of part (i) of this proposition, but now assume $x_i(\underline{v}, 0) = 1$. Then we have

$$p_{div,info}^*(0) = \underline{v} + \Delta \frac{\tau}{\lambda_{div}^* \tau + 1 - \lambda_{div}^*}$$

and

$$P_{div,info}(v_i, \tau) = \begin{cases} \lambda_{div}^* \underline{v} + (1 - \lambda_{div}^*) p_{div,info}^*(0) & \text{if } v_i = \underline{v} \\ p_{div,info}^*(0) & \text{if } v_i = \bar{v}. \end{cases}$$

Recall, in this case $\lambda_{div}^* = \frac{L/n}{(1-\tau)\underline{v}}$.

Moreover, consider the equilibrium under concentration with $\varepsilon = 0$. In this case,

$$\begin{aligned} x_i(v_i, \theta_i) &= \begin{cases} 0 & \text{if } v_i = \underline{v} \text{ and } \theta_i = 0 \text{ or, } v_i = \underline{v} + \tau\Delta \text{ and } \theta_i = 0 \\ \bar{x}_{con} \equiv n \times \min \left\{ \frac{L/n}{\bar{p}_{con}}, 1 \right\} & \text{otherwise,} \end{cases} \\ p_{con,info}^*(x_i) &= \begin{cases} \underline{v} + \Delta \frac{\tau}{\lambda_{con}^* \tau + (1 - \lambda_{con}^*)} & \text{if } x_i = 0 \\ \bar{p}_{con} = \underline{v} + \Delta \frac{\beta\tau}{\lambda_{con}^* (1 - \beta)(1 - \tau) + \beta} & \text{if } x_i \in (0, \bar{x}_{con}] \\ \underline{v} & \text{if } x_i > \bar{x}_{con}. \end{cases} \end{aligned}$$

and

$$P_{con,info}(v_i, \tau) = \begin{cases} \beta \bar{p}_{con} + (1 - \beta) [\lambda_{con}^* \bar{p}_{con} + (1 - \lambda_{con}^*) p_{con,info}(0)] & \text{if } v_i = \underline{v} \\ \beta \bar{p}_{con} + (1 - \beta) p_{con,info}(0) & \text{if } v_i = \bar{v}. \end{cases}$$

This implies that if $\lambda_{con}^* = \lambda_{div}^* = \lambda^*$ then

$$P_{div,info}(\bar{v}, \tau) > P_{con,info}(\bar{v}, \tau) \Leftrightarrow p_{div,info}^*(0) > p_{con,info}(0)$$

which always holds, and

$$\begin{aligned} P_{div,info}(\underline{v}, \tau) &< P_{con,info}(\underline{v}, \tau) \Leftrightarrow \\ \lambda^* \underline{v} + (1 - \lambda^*) p_{div,info}^*(0) &< \beta \bar{p}_{con} + (1 - \beta) [\lambda^* \bar{p}_{con} + (1 - \lambda^*) p_{con,info}(0)] \Leftrightarrow \\ \lambda^* \underline{v} + (1 - \lambda^*) p_{div,info}^*(0) &< \lambda^* \bar{p}_{con} + (1 - \lambda^*) [(1 - \beta) p_{con,info}(0) + \beta \bar{p}_{con}] \Leftrightarrow \\ \beta (1 - \lambda^*) (p_{con,info}(0) - \bar{p}_{con}) &< \lambda^* (\bar{p}_{con} - \underline{v}) \Leftrightarrow \\ \frac{1 - \lambda^*}{\lambda^* \tau + (1 - \lambda^*)} &< \frac{\beta (1 - \lambda^*) + \lambda^*}{\lambda^* (1 - \beta) (1 - \tau) + \beta} \Leftrightarrow \\ (1 - \lambda^*) \tau &> -\lambda^* \tau \end{aligned}$$

which always holds. Thus price informativeness is higher under diversification, if the investigation cutoff is the same. Suppose n is sufficiently large and $\beta < 1$ is sufficiently close to one such that $L/n \leq \Lambda(\beta) \cdot (1 - \tau)\underline{v}$ and $\lambda_{con}^* - \lambda_{div}^* \in (0, \varepsilon)$ (note that λ_{con}^* is invariant to n if $L/n \leq (1 - \tau)\underline{v}$) for some arbitrarily small $\varepsilon > 0$. Then, the above equilibrium under diversification has a strictly lower cutoff but strictly higher price informativeness than the above equilibrium under concentration, as required. ■