Kinetic order-disorder transitions in a pause-and-go swarming model with memory

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ARTICLE INFO

Keywords:
- Collective motion
- Agent-based models
- Metastability
- Phase transitions
- Memory
- Intermittent motion

ABSTRACT

A two dimensional model of self-propelled particles combining both a pause-and-go movement pattern and memory is studied in simulations. It is shown, that in contrast to previously studied agent based models in two-dimensions, order and disorder are metastable states that can co-exist at some parameter range. In particular, this implies that the formation and decay of global order in swarms may be kinetic rather than a phase transition. Our results explain metastability recently observed in swarming locust and fish.

1. Introduction

The question of when and how groups of moving animals can form large masses of coordinated swarms using only short-ranged, local interactions has been under intense investigation both experimentally and theoretically using computer simulations or analytic approximations. See (Vicsek et al., 1995) for a recent review. In particular, the original Vicsek model (Vicsek et al., 1995) showed that a change in system parameters, for example noise or density, can result in the particles clustering and synchronizing their direction. Moreover, the macroscopic behavior of the system can be divided into distinct phases in the sense of statistical physics with a phase transition occurring at a critical noise or density. A large number of generalizations and extensions of the Vicsek model have been suggested and studied, including, for example, different types of noise (Grégoire and Chaté, 2004), heterogeneous systems (Ariel et al., 2015), obstacles (Chepizhko and Peruani, 2013) and more. The main goal shared by these approaches is a characterization of the possible phases of the system at different parameter ranges.

On the other hand, recent experiments with schooling fish (Tunstrøm et al., 2013) and swarming marching locusts (Buhl et al., 2006; Yates et al., 2009; Ariel et al., 2014) have shown that the coarse grained dynamics of such moving animal collectives can be approximated by an effective stochastic differential equation (Yates et al., 2009; Kolpas et al., 2007). Under this description, the dynamics of the system quickly relaxes into one of the available metastable states and fluctuates around a local stable point. Transitions between the states are kinetic and can be considered as rare events. In the limit of low noise, the waiting time between transitions (also called the escape times) are expected to become independent and memory-less, i.e., have an exponential distribution. This type of dynamics is analogue to transitions between metastable states in systems following Langevin dynamics (Schlick, 2010). For example, Tunstrøm et al. (2013) identify three metastable states is small schools of fish (up to 300 individuals): a polarized state characterized by high average alignment, a milling state characterized by synchronized rotation and a so called swarming state in which both alignment and rotation are low. Similarly, it has been suggested that the dynamics of small swarms of locusts (up to 100 individuals) marching in a circular arena can also be classified into three metastable states corresponding to a clockwise or counter-clockwise moving swarm (Buhl et al., 2006; Yates et al., 2009) and possibly also a disordered state (Ariel et al., 2014).

These findings suggest two different points of view on the buildup and maintenance of order in collectively moving animals. The first considers order and disorder as distinct phases. Under this approach, a phase transition requires a change in parameters such as density, noise intensity or topology. On the other hand, the second approach regards transitions as dynamic while parameters (or the environment) only determine the admissible metastable states, escape times and transition rates. In particular, a swarm may switch between different disordered and ordered states spontaneously.

Numerical and analytical studies suggest that the difference between the two points of view depends on model details. In one-dimensional (1D) systems, an ordered phase of synchronized particles is trivially split into at least two metastable states of particles moving in a clockwise (CW) or counter-clockwise (CCW) directions. The main...
question relevant to the discussion here is whether the disordered state is also stable. The model of Czirok et al. (1999) describes simulations predicting a phase transition behavior. When noise is sufficiently large (or if the density is low), only a single stable disordered state exists. With lower noise (or higher density), two stable configurations corresponding to synchronization CW or CCW motion exist. Transitions between the two ordered states are kinetic (Ariel and Ayali, 2015). However, there is no coexistence between the ordered and disordered states. This result has been largely confirmed analytically (Garnier et al., 2016). Romanczuk and Erdmann (2010) study a related model of active Brownian particles in 1D both numerically and analytically using a mean field approximation. They find a bifurcation in the mean velocity allowing either only a disordered state, only ordered, or a mixed state, depending on system parameters. Finally, motivated by the motion pattern of marching locusts, Ariel et al study a variation of the Czirok model in which the movement of individual is not continuous. Instead, individuals can pause (stop moving). The trigger to resume movement and the direction of motion depends on the local dynamics around each focal particle (see details below). Numerical simulation show that, depending on system parameters, both ordered (CW or CCW) and disordered may be stable, conforming to the kinetic point of view.

The main purpose of this paper is to address and compare these two points of view (phase transition or kinetic switching) within a simplified two-dimensional (2D) model. This generalizes the one-dimensional (1D) model proposed in Ariel et al. (2014) to the more realistic 2D setting. Previous studies did not identify bistability of both ordered and disordered states in 2D either with constant speeds (Ariel and Ayali, 2015) or with varying ones (Romanczuk and Schimansky-Geier, 2012), which is in some sense similar to pause-and-go dynamics. We suggest a variation of the Vicsek model (Vicsek et al., 1995) by introducing two biologically realistic properties: an intermittent pause-and-go movement pattern and memory. Simulations show that the occurrence of kinetic switching between ordered and disordered states can be explained as a generic consequence of the interplay between these two effects. Loosely speaking, our model assumes a positive feedback between the propensity of an individual to move and the local order around it. This feedback can lead to the emergence of two stable configurations or states. The first is characterized by a high fraction of synchronized moving individuals. In the second state, most individuals are not moving (pausing). As a result, the effective density of individuals that are “participating in the dynamics” is low and the system is disordered. We show that with memory, both these states are stable to small perturbations. An accumulation of random fluctuations may build up to trigger a rapid transition to a different state.

The first new constituent of the model, intermittent pause-and-go dynamics, is a common movement pattern in many organisms (Kramer and McLaughlin, 2001). For example, some fish swim in a burst-and-coast movement pattern, presumably as an energy economy strategy (Fish, 2010). In Ginelli et al. (2015) it is shown that the movement of sheep in herds alternates between slow group dispersion and rapid regrouping events. Modeling this dynamics using a Vicsek-type 2D model with intermittent motion reveals that this behavioral pattern can have a significant effect of the group dynamics. Marching locusts, which are one of the primary motivation for this model, have also been shown to walk and stop intermittently - a pattern which has been suggested to be an important feature of locust locomotion (Ariel et al., 2014; Bazazi et al., 2012).

In our model, each animal (particle) is either standing (which we will loosely call sleeping), or moving (awake). Motivated by observations of locusts, particles can only change their orientation when waking up (switching from sleeping to waking state). As a result, movement is always in straight lines. We show that, in contrast to the 1D case (Ariel et al., 2014; Ariel and Ayali, 2015), this model does not produce robust kinematic order-disorder transitions. Then, we show that assuming particles have a simple form of memory, qualitatively changes the dynamics of the system to allow coexistence of metastable ordered and disordered states.

Most of the work on the subject of collective motion assumes for simplicity that animals or particles interact with the environment or with other particles instantaneously. However, memory can affect the collective behavior of animals in a variety of ways. For example, in Letendre and Moses (2013) it was shown that ants remember the location of food sources and that relying exclusively on the strength of pheromone trails, the number of ants foraging a single food patch will overshoot. By incorporating memory of past food sources into the foraging strategy the colony is able to gather more food. Experiments with glass prawns (Mann et al., 2013) swimming in an annular arena were shown to be best described by a model of Self-Propelled Particles (SPP) with memory in which the turning probability is a function of their current and past neighbors. The model successfully captures the macroscopic orientation of the whole school as well as the microscopic turning probabilities. Memory does not have to be an inherent property of the animals. It can be, as in ferromagnetic metals, as a result of the macroscopic state of the system. For example, Couzin et al. (2002) suggest a SPP model of fish moving in a three dimensional pool. They identify four types of collective movements patterns, depending on the range of the interaction. A hysteresis phenomenon was discovered whereby the current macrostate depends also on the recent history.

Memory effects can also improve the fit of SPP models to experiments. In many Vicsek-type models the noise changing a particle’s heading is an independent random variable, sampled at each time step. This results in a jerky movement. However, experiments on fish shoals show a smoothly changing direction (Gautrais al., 2009). A modification of the Vicsek model whereby the noise itself is a non-Markovian process (Nagai et al., 2015) captures this experimental result while producing many additional macrostates.

The layout of the paper is as follows. Section 2 briefly recalls the original Vicsek model and describes some of the analysis tools used to define and analyze the macroscopic states of the system. In addition, we present the pause-and-go model of motion. Section 3 introduces the pause-and-go model with memory. It is shown that this model induces a new behavior in which synchronized and unsynchronized states can be metastable. The dynamics of the system and the dependence on parameters is analyzed in simulations. We conclude in Section 4.

2. The vicsek model

Self-propelled particles are commonly used to study collective motion. The Vicsek model (Vicsek et al., 1995) is one of the first and simplest SPP models. It captures a collective motion phase stemming from the movement of many individual particles. The particles observe a simple short range interaction rule on their orientation, which results in close particles tending to move in unison. A random noise component is added to each particle’s heading, disturbing the synchronization. Vicsek et al. showed numerically that, with a fixed density and for increasingly large systems, a phase transition occurs at a critical noise level. Above the threshold noise the particles are barely aligned and are uniformly spread, while below the threshold they move in a single direction and are tightly clustered. Close to the critical noise, the order parameter exhibits power-law dependency on the noise and density of particles. This is consistent with a description of physical phase-transitions in which noise is, loosely speaking, an analogue of temperature. For a detailed discussion of effective temperature in Vicsek-type models see Ariel et al. (2015) and Porfiri and Ariel (2016).

2.1. Model description

Consider N particles moving in a square domain with sides of length L and periodic boundary conditions. All particles move at a constant speed v. A neighborhood of radius R defines the interaction distance. The location and velocity of particle i at time step t are
denoted \( x_i(t) \) and \( \xi_i(t) \), respectively, where \( x_i(t) \in [0, L]^2 \) and \( \xi_i(t) \) is the effective drift of all particles is computed according to the following equation:

\[
x_i(t + 1) = x_i(t) + v_i(t) \mod L.
\]

The updating process of the direction \( \xi_i(t + 1) \) of a particle is calculated in two stages: first, find the average direction of neighboring particles \( \hat{\xi}_i \), and then rotate by an angle \( \xi_i(t) \) that represents random noise. Denote the set of indexes of all particles interacting with \( i \) (including itself) at time \( t \) as \( I(t) = \{ j : \| x_i(t) - x_j(t) \| \leq R \} \). The average direction in the neighborhood of \( i \) is therefore

\[
\hat{\xi}_i(t) = \frac{\sum_{j \in I(t)} \xi_j(t)}{\| \sum_{j \in I(t)} \xi_j(t) \|}.
\]

The external noise is an independent identically distributed continuous uniform random variable that is dependent only on the noise intensity \( \eta \). It changes the direction of a particle via the two dimensional rotation matrix with angle \( \xi_i(t) \):

\[
\hat{\xi}_i(t) = R_{\xi_i(t)} \hat{\xi}_i(t), \quad R_{\xi_i(t)} = \begin{pmatrix} \cos(\xi_i(t)) & -\sin(\xi_i(t)) \\ \sin(\xi_i(t)) & \cos(\xi_i(t)) \end{pmatrix}
\]

Variable are updated synchronously, i.e., every simulation step the average orientation \( \langle \hat{\xi}_i(t) \rangle \) of all particles is computed according to the current locations \( x_i(t) \) and directions \( v_i(t) \). Then, the new locations and directions are updated for all particles.

The model exhibits a phase transition at a specific threshold noise or density \( \rho = N/L^2 \). Loosely speaking, below the threshold noise, particles point at a common direction, i.e. are synchronized. In order to quantify the synchronization in directions, define the instantaneous order parameter \( \psi(t) = \frac{1}{N} \sum_{i \in I(t)} \| \xi_i(t) \| N \). The global order parameter, also termed the polarization, is defined as \( \phi = \lim_{t \to \infty} \langle \psi(t) \rangle \), where brackets denote an average over initial conditions and realizations of noise.

### 2.2. Coarse grained analysis

The time evolution of the order parameter \( \psi(t) \) tracks the macroscopic state of the system (Bühl et al., 2006; Ariel et al., 2014). Yates et al. (2009) suggested to approximate the instantaneous order parameter behavior as a drift and diffusion process (see also Ariel et al., 2014; Erban et al., 2006; Haataja et al., 2004; Bode et al., 2010). Denoting by \( f(\psi, t) \) the probability distribution for the system to have an order parameter \( \psi \) at time \( t \), \( f(\psi, t) \) satisfies the Fokker-Planck equation

\[
\frac{df(\psi, t)}{dt} = -\frac{\partial F(\psi, t)}{\partial \psi} + \frac{\partial^2 D(\psi, t)}{\partial \psi^2}.
\]

where \( F(\psi) \) and \( D(\psi) \) are the effective drift and diffusion functions, respectively. The effective coefficients can be numerically approximated from the dynamics by

\[
F(\psi) \approx \left\langle \frac{\psi(t + \Delta t) - \psi(t)}{\Delta t} \right\rangle \psi
\]

\[
D(\psi) \approx \frac{1}{2} \left\langle \left( \frac{\psi(t + \Delta t) - \psi(t)}{\Delta t} \right)^2 \right\rangle \psi
\]

Here, \( \Delta t \) is a short time segment, on the time scale of the dynamics of \( \psi(t) \) (see Yates et al. (2009) and Ariel and Ayali (2015) for a discussion on the choice of \( \Delta t \)). For the rest of this paper, we take \( \Delta t = 1 \). The average \( \langle \cdot \rangle_\psi \) is calculated as follows: for a given range of \( \psi \) (and allowing long enough runs of the simulations such that the instantaneous order parameter visits this value many times) calculate the mean of \( \psi(t + \Delta t) - \psi(t) \) and continue as in the above formulas.

The drift-diffusion approximation is clearly a considerable simplification of the system dynamics. In particular, it does not account for the complexity of the spatial distribution and its impact on the order parameter (Romanczuk and Schimansky-Geier, 2012). However, it offers a convenient and relatively simple tool for identifying the metastable states in the system and the transitions between them. Under this approximation, metastable states are associated with the stable zeros of the effective drift \( F(\psi) \). A point \( \psi^* \), such that \( F(\psi^*) = 0 \) is called stable if a small positive perturbation of \( \psi \) will induce a negative drift, i.e. the expectation of the changes in the value of \( \psi \) will decrease, or oppose the direction of the perturbation, while for negative perturbations the drift will be increasing.

Fig. 1A depicts the time evolution of \( \psi(t) \) for two example values of the noise. Fig. 1B and C show the effective drift and diffusion functions as obtained from simulations. The simulation constants are detailed in Table 1.

### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Default value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Number of particles</td>
<td>4096</td>
</tr>
<tr>
<td>R</td>
<td>Radius of interaction</td>
<td>1</td>
</tr>
<tr>
<td>v</td>
<td>Speed</td>
<td>0.1</td>
</tr>
<tr>
<td>L</td>
<td>Square edge length</td>
<td>82.62</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Noise intensity</td>
<td>0.45 or 0.63</td>
</tr>
</tbody>
</table>
long times to a unique invariant (equilibrium) distribution denoted $\rho(\psi)$. This distribution depends on the evaluated functions $F(\psi)$ and $D\psi$. Fig. 1B and C compare this distribution with the density of $\psi$ obtained from a single long trajectory of the Vicsek model. Following Yates et al. (2009), $\rho(\psi)$ is given by

$$
\nu(\psi) = - \int_0^\psi F(s) \frac{ds}{D(s)} + \ln(D(\psi)), \quad \rho(\psi) = \frac{\exp(-\nu(\psi))}{\int_0^\psi \exp(-\nu(\psi)) d\psi} \quad (5)
$$

The function $\nu(\psi)$ is called the potential. Fig. 2 verifies that the drift-diffusion process provides an excellent approximation for $\nu(t)$. Typically, the potential tends to be lower close to stable fixed points of the drift and, as a result, the density $\rho(\psi)$ is higher. As a result, the number of stable fixed point is related to the number of local maxima in $\rho$. Moreover, if two stable fixed points are separated by a potential barrier that is large compared to the amplitude of the noise $\sqrt{D(\psi)}$, then the two states are called metastable. The reason is that trajectories will tend to stay in the area of a fixed point for long times until a rare large fluctuation moves the trajectory to a different metastable state. In this case, the waiting times between transitions are approximately independent and are expected to have an exponential distribution (Yates et al., 2009; Schlick, 2010).

2.3. Pause-and-go model of motion

In the original Vicsek model, particles are constantly moving. Motivated by the motion pattern of locusts and following Ariel et al. (2014), we study the dynamics of a new behavioral model in which particles stop moving occasionally in order to decide on a new direction. Accordingly, we assume that each particle is either “awake” or “asleep”. A particle can switch from one state to the other according to the feedback it receives from its immediate environment. Furthermore, a particle determines its direction of movement only when it wakes up. Similar to Vicsek, the new direction is set to be the local average direction plus noise. Moreover, if two stable fixed points are separated by a potential barrier that is large compared to the amplitude of the noise $\sqrt{D(\psi)}$, then the two states are called metastable. The reason is that trajectories will tend to stay in the area of a fixed point for long times until a rare large fluctuation moves the trajectory to a different metastable state. In this case, the waiting times between transitions are approximately independent and are expected to have an exponential distribution (Yates et al., 2009; Schlick, 2010).

To be precise, the state of a particle is given by a Boolean variable denoted by $w_i(t)$ describing whether the particle is awake ("go" state) or asleep ("pause" state). The direction is updated only if the particle is awake at the current time step and awakens at the next ($w_i(t) = \text{False}$ and $w_i(t+1) = \text{True}$). Awake particles increase the probability of their sleeping neighbors to wake up as well. Let $m_i(t)$ denote the fraction of awake particles in the neighborhood of particle $i$ at time $t$, $m_i(t) = \frac{|\text{set of awake neighbors}|}{|\text{set of i's neighbors}|}$ where $\text{set of awake neighbors} = \{j; j \in \text{set of i's neighbors} \cap w_j(t) = \text{True}\}$ is the set of awake neighbors (note that $|\text{set of i's neighbors}| \geq 1$ because a particle is its own neighbor). Absolute values denote the number of elements in a set. The probability of a particle to wake-up or fall asleep (i.e. $w_i(t+1) = \text{False}$) is determined as follows: if $m_i(t)$ is above a threshold value $M_{sw}$, then a sleeping particle wakes up with probability $P_{sw}$ per time step, otherwise, the probability is $P_{sw}(\psi)$. The probability to fall asleep is $P_{sw}(\psi)$. See Table 2 for the values of the constants used in simulations and Fig. 3 for a decision tree representation. A particle that wakes up takes the average direction of its awake neighbors. If there are none it is randomly (uniformly) chosen. A particle that falls asleep forgets its direction.

Fig. 4 shows the effective behavior of the 2D pause-and-go model without memory close to the critical value of $\eta = 0.265$. Even though the density of $\psi$ is bi-modal (Fig. 4C), the potential barrier between the disordered and ordered states is low (of the order of $10^{-4}$) while noise is large ($\sqrt{D(\psi)} \sim 10^{-3}$). As a result, this state cannot be considered as metastable. Indeed, trajectories do not appear to be bi-stable. Furthermore, the joint distribution of the order parameter and the fraction of awake particle (Fig. 4D) is uni-modal.

3. Pause-and-go with memory

The two dimensional pause-and-go model described above does not exhibit bistability as in the 1D case. An analysis of small perturbations reveals that in the case of the synchronized state, small sleeping clusters are easily swept away upon contact with a large awake cluster. In the unsynchronized state, small waves of awake particles quickly increase their distance, which results in a break-up of the cluster and as a result are easily effected by the ambient sleeping particles. A new property is needed to increase the stability of such clusters. This is done through the introduction of memory of past neighbors, which decreases the effect of small perturbations on a particle’s local environment. To this end, we assume that particles are affected by a weighted average of their current and past environment (within the interaction distance) rather than only their instantaneous neighbors. This results in higher stability and longer living active/sleeping clusters that can propagate across the system. For simplicity, we implement a memory kernel in which the weight of past events decays exponentially with time steps. This allows incorporating all the past history of a particle into a single variable without requiring explicit tracking of all past neighbors. We note that the process is still Markovian. More complicated memory kernels may affect the approximation as a Markovian drift-diffusion process and therefore also the distribution of escape times from metastable states.

In order to incorporate past values of the fraction of awake

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Default value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{sw}$</td>
<td>Fraction of walking particles threshold</td>
<td>0.377</td>
</tr>
<tr>
<td>$P_{sw}$</td>
<td>Probability to fall asleep</td>
<td>0.03</td>
</tr>
<tr>
<td>$P_{sw}$</td>
<td>Probability of waking above threshold</td>
<td>0.136</td>
</tr>
<tr>
<td>$P_{sw}$</td>
<td>Probability of waking below threshold</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Fig. 3. Decision process describing a particle's sleep-awake state. The decision tree describes how particles determine whether to wake-up or fall asleep. The process is a random variable that depends on a particle’s current state $w_i(t)$ and fraction of awakened neighbors $m_i(t), U(0, 1)$ denotes an independent sample of a uniform random variable in $[0, 1]$. $P_{sw}$, $P_{sw}$, $P_{sw}$, and $M_{sw}$ are constants (see Table 2).
neighbors, we define a new state variable \( m_t \), which is a weighted average of the current \( m_{t-1} \) and the previous \( m_{t-2} \). It is updated according to,

\[
m_t = \frac{1}{2} m_{t-1} + \frac{1}{2} m_{t-2}
\]

where \( S \in [0, 1] \) determines the decay rate. The characteristic memory time is \( \tau = -1/\log S \). If \( S=0 \) we regain the behavior of the previous memory-less system, while if \( S=1 \) then the particle has a constant \( m \) that is set in the initial condition and is independent of time. Overall, the internal state of each particle is described by four time dependent properties as summarized in Table 3.

### Table 3

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
<th>Update rule</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x(t) )</td>
<td>Location</td>
<td>( x_t = x_{t-1} + v_t(t-1) \mod L )</td>
<td>Eq. (1)</td>
</tr>
<tr>
<td>( v(t) )</td>
<td>Direction</td>
<td>( v_t = R_t \xi(t-1) )</td>
<td>Eq. (2)</td>
</tr>
<tr>
<td>( m_t )</td>
<td>Memory of neighbors</td>
<td>( m_t = m_{t-1} + (1 - S)m_{t-1} )</td>
<td>Eq. (6)</td>
</tr>
<tr>
<td>( w(t) )</td>
<td>Awake or asleep flag</td>
<td>Decision tree</td>
<td>Fig. 3</td>
</tr>
</tbody>
</table>

![Fig. 4. Dynamics in the pause-and-go model (without memory). The effective drift (A) and diffusion (B) in the order parameter close to the critical point, \( \eta = 0.265 \). Filled (empty) circles show the stable (unstable) fixed points. (C) The density of the order parameter, \( \psi \). (D) The joint density of the order parameter and the fraction of awake particles. Even though the density of \( \psi \) is bi-modal, the stability of the disordered state is low because the potential barrier (the local minimum of \( F(\psi) \)) is smaller than the noise level \( \sqrt{D\psi} \). Furthermore, the joint distribution is uni-modal.](image)
lower values of the chosen parameter values. Fig. 6 shows the drift for 12 different configurations. The two metastable states are consistent across all configurations. The critical noise, which depend on the system parameters are $\eta = 0.23, 0.237$ (A), 0.2437, 0.212 (B), 0.225, 0.235 (C), 0.226, 0.2426 (D), 0.212, $-0.2365$ (E) and 0.241, 0.2 (F).

### 3.1. Low noise – order/disorder bistability

The addition of memory qualitatively changes the model dynamics. Fig. 7 depicts the time evolution of the order parameter and the fraction of moving particles, $f(t) = |\psi(t)|$, $w(t) = True|/N$. The dynamics clearly shows fluctuations around two distinct values with low or high synchronization. Moreover, the fraction of awake particles and the instantaneous order parameter are highly correlated (Pearson coefficient 0.96). The figure also shows classification into ordered and disordered states.

Applying the coarse grained analysis described in Section 2.2, the effective drift and diffusion functions can be approximated from simulations. Fig. 8A and B show three zeros for $F_\psi$. Two fixed points are stable, corresponding to the ordered and disordered states. Fig. 8C and D show the marginal density of the order parameter and the joint density of $\psi$ and $f$, indicating the bi-stable dynamics and the correspondence between $\psi(t)$ and the fraction of awakened particles. In addition, Fig. 8C compares the density of the order parameter obtained by rough equal time spent in the two metastable states. Each graph corresponds to a different parameter changed, while the others remain at the default values. (A) concentration (B) probability of waking below threshold (C) probability to fall asleep (D) fraction of walking particles threshold (E) probability of waking above threshold (F) memory strength.
by a single long trajectory with the one obtained by the effective drift and diffusion in the effective drift-diffusion process.

Further examination of the dynamics during a transition between the metastable states shows that synchronization typically builds up as a cluster of particles that is moving parallel or in diagonal to the boundary domain and forms a wave. Such wave phenomenon have been seen in the original Vicsek model (Grégoire and Chaté, 2004) and other variants (Nagai et al., 2015; Chaté et al., 2008). They are effected by the interaction with the shape of the periodic boundary (Nagy et al., 2007) and are dominant when the speed of the particles is high (i.e. moving a large amount of distance without changing direction). This phenomenon is more pronounced in the pause-and-go model because particles change direction less frequently. Waves seem to have a sharp front with an exponentially decaying tail and are composed of many smaller waves (Chaté et al., 2008). Fig. 9 shows snapshots from each state and typical transitions. Disordered states are characterized by a uniformly random distribution of particle orientation and slightly clustered distribution of positions with no apparent structure. Almost all particles are asleep in this state (Fig. 9A). Due to fluctuations, a small number of spontaneously awakened particles can occasionally form a wave-like cluster of awake individuals (Fig. 9B). The positive feedback in the probability of particles to wake up increases the chances that other particles in the vicinity of the wave wake up as well and move in the same direction. The ordered state in Fig. 9C has the form of a density wave whose shape and orientation depends on the boundaries of the computational domain. Finally, Fig. 9D shows a
transition to the disordered state. Clusters in which the majority of particles are asleep are highly stable. A sufficiently large sleeping cluster can break the high density waves. These large dense clusters are formed only in the ordered state due to the increased number of awake particles, which increases the number of particles further absorbed into the clusters. Few particles typically leave the cluster as they have a very low chance to wake-up. This is because their current and past environment is composed of only sleeping particles. Such a cluster would spread and eventually break-up in the disordered state because the particles have a slight probability to wake-up regardless of their environment and their random movement will spread the cluster until the particles are not within the interaction radius. In other words, the main effect of memory is to increase the stability of large clusters of sleepers. This results in a decrease of the critical noise.

Escape times quantify the duration that trajectories stay around a metastable state before switching to a different one. We assume that the simulation is in a synchronized state when \( \psi \) is above a predetermined threshold value and unsynchronized when it is below a (lower) threshold. The duration between the two states is divided equally. The escape times for a metastable state that results from a drift-diffusion process are expected to be independent and approximately follow an exponential distribution. Fig. 10 shows good agreement between simulation data and theory – once transition times are subtracted (around 5000 steps), the distribution of escape times is approximately exponential.

Fig. 11 shows the effects of the number of particles on the stability of each state and the critical noise. The critical noise is defined as the noise value such the amount of time in either state is roughly equal.
The fit is consistent with standard finite sized scaling analysis (Baglietto and Albano, 2008) where the relative critical noise increases as a power law in $L$. The increase in escape times from the disordered state is explained by the fact that a small perturbation of awake particles in the disordered state must reach a size comparable to $L$, however, as the wave becomes larger and longer living, the more its particles remember they are in an active environment (i.e., have a higher value of $m$ and less chance to fall asleep). Escape times from the ordered state require a formation of a large sleeping cluster. However if such a cluster is formed then the number of awake particles decreases - which decrease the flux of particles into the cluster and further increase in cluster size is discouraged.

3.2. High noise – disordered state

When the noise is increased the order parameter and escape times from the synchronized state decrease, while the escape times from the unsynchronized state increase. Above the critical noise the simulation can not remain synchronized and we regain the classical behavior of a single stable disordered state (Fig. 12).

4. Discussion

The main motivation for this paper is the fundamental question of how collective motion of short-ranged interacting animals is formed. We have presented a two-dimensional model of moving animals or
organisms which involves two separated time scales - an instantaneous short-ranked interaction rule and simple memory of past encounters. In addition, based on experimental observations of fish and locusts that follow an intermittent motion pattern, particles in our model can switch between two different levels of activity (sleep or awake) that are also determined by the same two scales. The dynamics in the new model is in stark contrast to the classical, physical phase transition observed in many similar SPP models, in which transition between states can only be achieved by a change in a global system parameter, such as average density or noise. Instead, we have shown that, at least for large but finite systems, the dynamics of the pause-and-go model with memory can be approximated by an effective drift-diffusion process with two metastable states corresponding to ordered and disordered states. For example, Calovi et al. (2014) suggest a biological model for the schooling of fish where the model hinges on the interplay between the intermittent pause-and-go dynamics and memory. We find that both are necessary to achieve this phenomenon.

It is important to note that in the limit of an infinitely large system, the dynamics may not be bistable, as can be seen from the increasing escape times depicted in Fig. 11B. Indeed, the observed metastability may be evidence of a first order phase transition (Baglietto et al., 2015). However, the infinite system limit is not necessarily the relevant one for many naturally occurring animal schools or swarms in which finite size effects may be of importance.

Of course, the model proposed in this article is not the only way to achieve metastability and kinetic switching between ordered and disordered states. For example, Calovi et al. (2014) suggest a biologically realistic model for the schooling of fish. The model incorporates several fine details which are specific to fish such as a frontal preference in the response of a focal individual to its neighbors. It is shown that in some parameter range the dynamics exhibits metastability and transitions between polarized and milling states. Our work suggests that this coarse-grained behavior of swarms may be a generic property of many natural systems showing collective motion. In particular, it does not depend on species-specific interactions or behavioral rules.

References