Asian carp includes four carp species: black carp (Mylopharyngodon piceus), grass carp (Ctenopharyngodon idella), silver carp (Hypophthalmichthys molitrix), and bighorn carp (Aristichthys nobilis). These species are spreading within the Mississippi River Basin and are causing environmental and economic perturbations. Although Asian carp are preferential lake-dwelling fishes, they require large rivers to spawn and support egg and larvae development. Both the drifting eggs and early larvae are semi-buoyant and it is believed they must remain suspended in the water column to survive.

- \( V_z \): is the eggs terminal fall velocity; \( u_t \): is the Von Karman's constant and \( z_r \): is the roughness height. The hydrodynamic fields (velocity, horizontal diffusion, and water depth) are discretized by cells in series.
- \( \nu = R_e \frac{2}{Re} \frac{C_p}{2} \) Where, \( R_e = \frac{C_p}{\nu} \) is the von Karman drag coefficient, \( C_p \) is the egg drag coefficient, \( \nu \) is the water depth, \( \beta \) is a factor that describes the difference in the diffusion of a discrete egg and the diffusion of a fluid particle. \( \beta = 1 + \frac{2}{\nu} \) for 0.1 < \( \nu \) < 1 Van Rijn (1998).
- \( K_z \): is the horizontal turbulent diffusivity, \( K_{\nu} \) : is the eddy diffusivity. \( K_z = 0 < K_{\nu} \), \( K_{\nu} = \beta V_z \) is the eddy diffusivity. Using the parameter eddy viscosity equation \( \nu^* = K_{\nu} (1 - \beta) \) The position of the eggs due to advection is solved deterministically using Eulerian forward solution for time stepping. The displacement of the eggs due to turbulent diffusion is simulated using a random component. The three dimensional position of each particle is adjusted according to the terminal fall velocity and the velocity and diffusion fields. The vertical movement of virtual eggs is simulated using the Random Displacement Method (Visser, 1997).

\[
\begin{align*}
X_n^{k+1} - X_n^k &= V_n k + R_n \\
Y_n^{k+1} - Y_n^k &= V_n k + R_n \\
Z_n^{k+1} - Z_n^k &= V_z k + R_z k
\end{align*}
\]

Where, \( X_n \) and \( Y_n \) are the 3D positions of the particles (eggs) at time \( t+1 \) and \( X_n^k \) and \( Y_n^k \) are the previous positions of particles at time \( t \). \( V_n \) is the fluid particle velocity. \( R_n \) is a random draw from a normally distributed random variable having mean zero and standard deviation one (Valocchi & Quinodoz, 1989).

- \( K_{\nu} = \beta V_z \): Non random advective component from low to high diffusivity areas. Represents the gradient of diffusivity
- \( K_z = 0 < K_{\nu} \): The diffusivity is estimated at an offset distance \( \alpha K_{\nu} / \beta V_z \) \( Z_n^k \) of the initial particle location \( Z_n^k \)
- When the vertical turbulent diffusivity is constant, the random displacement model is identical to the random walk model.

Reflective boundaries were included in the model. If a particle position exceeds the boundaries of the study channel, the particle is placed back in the model domain at a distance that is equal to the distance that the particle exceeds the boundary.

**3. Mathematical Model**

Governing equation of the 3-dimensional mass transport of Asian carp eggs neglecting sink and sources:

\[
\frac{\partial n}{\partial t} + \nabla \cdot (V_n n) + \nabla \cdot (\kappa_n \nabla n) = \frac{\partial n}{\partial t} + \nabla \cdot (V_n n) + \nabla \cdot (\kappa_n \nabla n) + \frac{\partial \kappa_n}{\partial t}
\]

\( n \) is a quantity that describes the amount of eggs; \( V_n \), \( V_z \), and \( V_t \) are the water velocity in the x,y, and z direction.

\[
V_t = \frac{2 V_z + \nu}{\nu}
\]

Smooth Boundaries

\[
V_t = \frac{2 V_z + \nu}{\nu}
\]

Rough Boundaries

\[
V_t = \frac{2 V_z + \nu}{\nu}
\]

**4. Case study: Flume experiments**

The case study used to validate FluEgg was based on the experiments conducted by (Tang et al. 1989). The experiments were performed in a 30 m long, 0.72 wide flume with a water depth of 0.6 m and a water temperature of 20 °C. The egg’s vertical concentration distribution was measured using an egg collector sampling device consisting of a series of layers connected to a series of rectangular nets. Grass carp egg vertical concentration distributions reported for 0.4 m/s flow velocities were used to validate FluEgg.

**5. Acknowledgements**

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