Spatial eigenmodes conversion with metasurfaces engraved in silicon ridge waveguides

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We explore the discrete nature of waveguide modes and the effective medium concept to achieve an ultra-compact highly-efficient mode conversion devices in a high index platform such as silicon waveguide. The proposed device is based on a co-directional coupler which has a periodic variation in its refractive index along the propagation direction. The transverse variation of the index profile is calculated based on the interference pattern between the modes of interest. We show that the mode conversion can be realized with dielectric metasurfaces engraved in silicon waveguide. We derive the equation for effective index and show proof-of-concept numerical results of the device performance. We obtain conversion efficiencies of 95.4\% between the TE\textsubscript{0}-TE\textsubscript{1} modes over 8.91 µm interaction distance and 96.4\% between the TE\textsubscript{0}-TE\textsubscript{2} over 6.32 µm, respectively. The resulting coupling coefficient changes as a function of the interaction distance in a sinusoidal manner which is crucial for constructive energy transfer from one mode to another. Such mode coupling devices have the potential for application in dispersion compensations, wavelength division multiplexing systems, and sensing.  © 2019 Optical Society of America

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1. INTRODUCTION

Photonic integrated circuits (PICs) based on Silicon-on-Insulator (SOI) technology hold a great potential in various applications such as dispersion engineering [1, 2], on-chip information processing [3], chemical and biological sensors [4, 5] and photonic multiplexing systems [6, 7]. However, a reliable, large-scale system integration of such a device still presents many challenges including miniaturizing device footprints, increasing device operation bandwidth and robustness, and reducing device losses.

Since the bandwidth of photonic systems is limited by their optoelectronics, advanced photonic multiplexing methods may provide a solution for the growing need for high data rates. These techniques allow for multiple optical signals to be transmitted simultaneously in the same channel. The signals are differentiated based on their wavelengths [8], polarization [9], position [10] or mode profile [6] and thus, increasing the system data capacity.

The mode-division-multiplexing (MDM) method [11] employs the orthogonality principle of the different spatial eigenmodes supported by a waveguide to encode data over each separate channel. In addition, it allows for multiplexing the information over a single multimode waveguide. A key element in a MDM system is a mode converter which ideally can couple any given spatial mode into any other mode.

Conventionally, the design of mode converters relies on techniques involving integration of multiple materials [12] and complex waveguide geometries [13–15], and supports conversion between only two specific modes [16, 17]. Here, we take advantage of the discrete nature of waveguide modes [18] and the effective medium concept [19] to allow the design of ultra-compact and high-efficient mode conversion devices in a high index platform such as sili-
waveguide. Furthermore, we do not use materials other than silicon for the device and conversion into different spatial modes is possible on the same waveguide. The proposed device is based on an all-dielectric co-directional coupler with a periodic index perturbation along the propagation direction. The transverse variation of the index profile is calculated based on the interference pattern between the modes of interest and mapped according to EIM to form a metasurface. Metasurfaces are artificial materials with subwavelength features which act as miniature, anisotropic light scatterers. As a result, the phase, amplitude and polarization of light can be engineered to provide the desired optical response. Our proposed waveguide mode converter can be scaled to realize arbitrary waveguide mode conversion. As an example, we demonstrate two silicon waveguide mode converters employing all-dielectric metasurface structures that can convert TE0 mode to TE1 and TE2 modes on the same waveguide, respectively.

2. THEORY AND DESIGN

A. Coupled Mode Theory

The mutual interaction between the propagating modes in an optical waveguide can be described by coupled mode theory (CMT) [18]. A finite and discrete number of orthogonal propagation modes exist in a waveguide and in the presence of a change in the waveguides cross section, optical modes can either couple or interfere with each other.

Our analysis is based on a 2D representation of the multimode waveguide. The electric field in the perturbed multimode waveguide can be written using the superposition of all the possible $E_m(x,y)$ eigenmodes in the unperturbed waveguide modes as

$$E(x,y,z,t) = \sum_{m=1}^{M} A_m(z)E_m(x,y)e^{i(\omega t - \beta_m z)}$$

(1)

where $m$ is the mode order, $A_m(z)$ is the complex field amplitude (CFA), $\beta_m$ is the propagation constant of the $m$th guided mode in the propagation direction and $\omega$ is the angular frequency.

The transverse wavefunctions $E_m(x,y)$ satisfy the unperturbed wave equation and form an complete orthonormal set such that

$$\int \int E_n^*(x,y)E_m(x,y)dx dy = \frac{2\omega \mu}{|\beta_m|} \delta_{mn}$$

(2)

We now consider the effect of a dielectric perturbation $\Delta\varepsilon(x,y,z)$. In the proposed device the perturbation is implemented as a weak perturbation to the index of refraction $n_a$ of the core of the multimode waveguide. The spatial dielectric tensor can be written as

$$\varepsilon(x,y,z) = \varepsilon_a(x,y) + \Delta\varepsilon(x,y,z)$$

(3)

where $\varepsilon_a$ is the unperturbed part of the dielectric tensor and $\Delta\varepsilon(x,y,z)$ represents the periodic dielectric perturbation along $z$ direction. We further assume that the dielectric perturbation is weak compared to the index of the waveguide core so that the variation of the mode amplitudes satisfies the slowly varying amplitude (SVA) approximation [20]

$$\frac{d^2}{dz^2}A_m \ll \beta_m \frac{d}{dz}A_m$$

(4)

Since the perturbation $\Delta\varepsilon(x,y,z)$ is chosen to be periodic in $z$, we can expand it into Fourier series

$$\Delta\varepsilon(x,y,z) = \sum_{k \neq 0} \Delta\varepsilon_k(x,y)e^{-ik(\frac{2\pi}{\beta_m - \beta_n})z}$$

(5)

where $\Delta\varepsilon_k(x,y)$ is the $k$th Fourier coefficient of the perturbation and period $\delta$ is determined by the phase matching condition given by

$$\delta = \frac{2\pi}{\beta_m - \beta_n}$$

(6)

This is the fundamental condition where coupling between two modes can occur. It is chosen as to compensate for the propagation constant mismatch of the two modes.

Solution of the wave equation using the expressions provided above consists of a set of coupled linear differential equations according to the number of all possible eigenmodes $M$

$$\sum_{m} \frac{d}{dz}A_m = -j\sum_{k \neq n} \sum_{m} \delta_{mn}(\beta_m - \beta_n - k2\pi/\delta)z$$

(7)

where $A_m(z)$ and $A_n(z)$ are the normalized complex mode amplitudes (CFA) along the propagation direction $z$, $\beta_m$ and $\beta_n$ are the propagation constants of the modes with transverse normalized electric field profiles $E_m(x,y)$ and $E_n(x,y)$.

These coupled mode equations relate the amplitudes of each possible modes in the waveguide in
terms of the phase mismatch and coupling coefficients $\kappa_{mn}^{(k)}$ defined as
\[ \kappa_{mn}^{(k)} = \frac{\omega}{4} \int \int E_m^*(x, y) \Delta \varepsilon_k(x, y) E_n(x, y) dxdy \] (8)

$\kappa_{mn}^{(k)}$ reflects the magnitude of the coupling between the $m$th and $n$th modes due to the $k$th Fourier components of the dielectric perturbation $\Delta \varepsilon(x, y, z)$.

The coupling can be explained as follows. According to Equation Eq. (7), the increment in the field amplitude of the $m$th mode $dA_m$, due to the coupling with the $n$th mode in the region between $z$ and $z + dz$ via the $k$th Fourier component of the dielectric perturbation.

In principle, a multiple and finite number of modes determined by the boundary conditions, material dispersion, and structure are involved. However, in practice, only two modes are strongly coupled and Eq. (7) reduces to a set of two equations
\[ \frac{d}{dz} A_m(z) = -j \kappa_{mn} A_n(z) e^{j(\beta_m - \beta_n)z} \] (9)
\[ \frac{d}{dz} A_n(z) = -j \kappa_{mn} A_m(z) e^{j(\beta_m - \beta_n)z} \]
where $\kappa_{mn}$ is given by Eq. (8) and the reciprocity relation of the coupling coefficients is $\kappa_{mn} = -\kappa_{nm}$. Solving the coupled mode equations in Eq. (9) in a case of co-directional coupler ($\beta_m > 0, \beta_n > 0$) and perfect phase matching, we obtain
\[ A_m(z) = [aA_m(0) + bA_n(0)] e^{-j\frac{\Delta \beta}{2}z} \] (10)
\[ A_n(z) = [a^*A_n(0) + bA_m(0)] e^{j\frac{\Delta \beta}{2}z} \]
where
\[ a = [\cos(qz) + j\frac{\Delta \beta}{2q} \sin(qz)] \] (11)
\[ b = -j \frac{\kappa_{mn}}{q} \sin(qz) \]
and
\[ q = \sqrt{\kappa_{mn}^2 + \left(\frac{\Delta \beta}{2}\right)^2} \] (12)
this solution would reveal a periodic exchange of power between the modes along the propagation.

**B. Design Principles**

In order to efficiently couple between the $m$th and $n$th modes in a waveguide, an effective wavevector $k_{ef}$ must be provided by the metasurface to overcome the propagation constant mismatch. Assuming a sinusoidal perturbation, the period of the structure which allows phase matching is given by Eq. (6). However, choosing constant perturbation based on the phase matching condition alone will cause the integral in Eq. (8) to vanish. Therefore, the perturbation $\Delta \varepsilon(x, y, z)$ must vary along $x, y$ in order to maximize the integral in Eq. (8) coupling coefficient. Furthermore, according to the phase matching condition, the modes will be out of phase after propagation over $\delta/2$. Therefore, changing the sign of $\kappa_{mn}$ from positive to negative value is required after $\delta/2$ to ensure constructive contribution to the conversion.

Similar to the formation of a hologram by the interference of two waves, we consider the interference of guided modes $m$ and $n$. The interference pattern between the $m$th and the $n$th eigenmodes is given by
\[ E_m(x, y)E_n(x, y)^* e^{-j(\beta_m - \beta_n)z} + c.c. \] (13)

Based on Eq. (13), we choose a sinusoidal perturbation in the form
\[ \Delta \varepsilon(x, y, z) = \epsilon_0 n_{Si} \Delta n(x, y, z) \]
\[ \Delta n(x, y, z) = \Delta n(x, y) e^{-i\delta z} \] (14)
\[ = C_0 E_m^*(x, y)E_n^*(x, y)e^{-i\beta z} \]
where $C_0$ is the amplitude of the index modulation. It is optimized through simulations in order to achieve maximal coupling efficiency over the shortest interaction distance.

Substituting Eq. (14) into Eq. (8), the coupling coefficient becomes
\[ \kappa_{mn}(z) = \frac{\omega \epsilon_0 n_{Si}}{2} e^{-i\delta z} \int \int C_0 |E_m(x, y)|^2 |E_n(x, y)|^2 dxdy \] (15)

The devices are reciprocal in terms of launching the fundamental input mode from either side of the waveguide. However, since the metasurface provides a wavevector in the opposite direction to the input, it provides a decrement in the propagation constant. Therefore, conversion from higher order to lower order modes is possible.

**3. RESULTS AND DISCUSSION**

Studied system is shown in Figure 1. Conversion of waveguide modes requires preliminary information about the propagation of the eigenmodes supported by a given waveguide. Figure 2b shows the cross section of the proposed SOI waveguide.
Fig. 1. Schematic configuration (side view) of the proposed mode converter composed from the profiled refractive index as tilted periodic slits: (a) $L$ is the interaction distance, $t$ is the metasurface thickness, (b) $\Lambda$ is the metasurface period achieved from the simulation and $\theta$ is the inclination angle which is $\theta = \tan^{-1}(2W/((j+1)\delta))$.

strip waveguide. In our design we use a waveguide with thickness of the top silicon layer ($n_{Si}=3.476$ [21]) of $H=200$ nm, strip width of $W=1400$ nm on top of a buried oxide layer ($n_{SiO_2}=1.444$ [21]) operating at wavelength $\lambda = 1.55$ $\mu$m. To calculate the modes supported by the waveguide, we used the Finite-Difference-Eigenmode (FDE) solver Lumerical MODE. This structure is a multimode waveguide that supports three TE$_i$ modes which have dominant electric fields in $\hat{y}$ direction. Figure 2a shows the $|E_y|^2$ component of the fundamental TE$_0$ mode which we use as an input to the device. We aim at converting this fundamental mode into higher order modes of the same polarization shown in Figures 2c (TE$_1$) and 2d (TE$_2$), respectively.

Fig. 2. Calculated normalized $E_y$ component the TE modes and structure of the proposed SOI waveguide: (a) fundamental TE$_0$ mode, (b) schematics cross-section of the studied waveguide, (c) TE$_1$ mode and (d) TE$_2$ mode.

Figure 3 shows the 3D-FDTD simulation results of a TE$_0$-TE$_1$ mode converter. The calculated refractive index profile depicted in Figure 3c is a sinusoidal modulation in both the longitudinal and transverse directions. The longitudinal period is determined based on the phase matching condition whereas the transverse profile is calculated based on 2D representation of the modes and extension of their interference pattern along the propagation direction [22]. The refractive index profile depends on the symmetry of the output mode. Odd modes will produce a symmetric profile while even modes will result in an asymmetric profile. The arrows in Figures 3a and 3b indicate the direction of the power flow.

The TE$_0$ mode in Figure 3a is launched at the input of the waveguide and propagates with no interruption. At the point where the perturbation starts, the mode couples and interferes with the TE$_1$ mode as a result of the index in Figure 3b, resulting in energy exchange between the modes 3d and the converted output mode shown in Figure 3b. Decomposition of the computed field in the basis of the waveguide eigenmodes allows us to determine the power transmission and crosstalk of each mode through the device [22]. Consequently, the conversion efficiency between the modes reaches 95.4% after only a single period resulting in device length of $L=8.91$ $\mu$m. The modulation index, $C_0=0.2$ for this case.

Fig. 3. Calculated results of TE$_0$-TE$_1$ ($E_y$ component) mode conversion device: (a) input and (b) output mode profiles, (c) refractive index profile required for conversion and (d) mode evolution along the propagation direction.

Similarly to the results presented above, we now
discuss the design considerations of a TE0-TE2 converter. Figure 4c shows the calculated refractive index profile. The profile is symmetric in the x direction since the desired output mode is odd. The treatment when considering conversion into an odd mode is similar to the previous case of an even mode. Figure 4a shows the input TE0 which is incident upon the waveguide with the metasurface section, the energy is coupled out of the input mode and into the output mode gradually along the section with the perturbation (Figure 4d) and finally outputs as TE2 mode (Figure 4b). The direction of power flow is depicted as arrows in Figure 4a and 4b. In this case the conversion efficiency is 96.4% between the TE0-TE2 over 6.32 µm of interaction with modulation index of C0=0.31.

Fig. 4. Calculated results of TE0-TE2 (Ey component) mode conversion device: (a) input and (b) output mode profiles, (c) refractive index profile required for conversion and (d) mode evolution along the propagation direction.

As expected, the FDTD simulations show behavior of power exchange between the modes. The refractive index profiles can be implemented as surface relief corrugations designed according to the effective medium theory (EMT). The device can be manufactured with different CMOS fabrication processes used in Nanoelectronics and Silicon Photonics such as electron beam lithography [23, 24], UV lithography [25], ionic implantation [26] and others [27–29].

Figure 5 shows the conversion efficiency and intermodal crosstalk between the modes as a function of wavelength at the output of the waveguide. The crosstalk with the input mode (TE0) in the TE0-TE1 converter reaches minimal value around the operation wavelength of λ=1.55 µm while the third TE mode supported by the waveguide (TE2) is suppressed throughout the simulated spectral range. As a result, the conversion efficiency is the highest at the operation wavelength and reaches 95.4%. The conversion efficiency and modal crosstalk of the TE0-TE2 mode converter shown in Figure 5b presents similar spectral behavior. The power of input mode is at its minimum at λ=1.55 µm and the TE1 mode is almost completely suppressed. As a result, the conversion efficiency reaches 96.4%.

Fig. 5. Calculated conversion efficiency and modal crosstalk of the mode converters: (a) TE0-TE1 and (b) TE0-TE2.

4. CONCLUSION

To conclude, we demonstrated the mode conversion of different spatial eigen-modes on an SOI waveguide. In all examples throughout this paper we have launched the TE0 mode to the input waveguide and built 3D FDTD simulation to compute the electric field at the output. Compared to conventional mode-conversion devices, we converted the modes on the same waveguide using only the waveguide material. The high conversion efficiency of the devices is achieved at a relatively short interaction length. This approach of engraving metasurfaces in a material is technologically simpler compared to fabricating metasurfaces in traditional overlayer on a waveguide as in ref. [30]. We showed that all-dielectric on-chip multimode conversion device can convert between TE0, TE1 and TE2 modes in silicon ridge waveguide and can be further generalized for higher
order modes. Our approach, may open up the horizons of novel chip-scale telecommunication devices such as switches, multiplexers and many others.

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